Optimal Insurance With Divergent Beliefs

About Insurer Total Default Risk

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Abstract

This paper extends the classic expected utility theory analysis of optimal insurance contracting to the case where the insurer has a positive probability of default and the buyer and insurer have divergent beliefs about the probability distribution of total default risk. The optimal marginal indemnity above the deductible is greater (smaller) than one if the buyer's assessment of default risk is more optimistic (pessimistic) than the insurer's. As an application of the model, we consider the market for reinsurance against catastrophic property loss and propose an expected utility theory explanation for the increasing and concave marginal indemnity schedule observed in this market.

Key words: catastrophe, insurance, default risk, risk perception

JEL Classification: D81

1. Introduction

Optimal insurance contract design has been studied extensively using the expected utility paradigm (see, for example, Arrow 1963, Mossin 1968, Raviv 1979, and Picard 2000).[1] Standard results are that risk-averse decision makers will fully insure if coverage is available at actuarially fair rates and that full insurance above a deductible is optimal when higher-than-actuarial prices are charged. With rare exceptions, these analyses have been conducted under the assumption that claims will always be paid, i.e., that the probability of insurer default is zero. However, in real-world insurance markets, insurers can and do become insolvent, resulting in non-payment or partial payment of claims (Cummins, Grace, and Phillips 1999, Cummins, Doherty and Lo 2002). It is not clear whether the standard optimal insurance results hold in a market with non-zero insurer default risk.

The objective of this paper is to analyze the design of an optimal insurance policy under the assumption that insurers have non-zero probabilities of default. We analyze a market with a risk-averse buyer and a risk-neutral insurer. Both the buyer and the seller are aware that the probability of insurer default is non-zero. However, since such an event is uncertain in a Knightian sense, the economic agents may have different perceptions of the insurer's default risk. Full insurance above a deductible is shown to be optimal when both parties have the same perception of the insurer's default risk, paralleling the standard theoretical result. However, when perceptions of default probabilities differ, optimal insurance coverage can be quite different from the standard results of insurance demand theory and depend upon the degree of risk aversion of the buyer and the relationship between the buyer's and the insurer's perceived probability distributions of insurer default. The principal objective of the paper is to show how the standard optimal insurance results change in the presence of asymmetrical information about insurer default rates.

As an application of the model, we consider the market for reinsurance against catastrophic events such as hurricanes and earthquakes. In this market, insurers with exposure to catastrophe event risk purchase reinsurance to hedge the risk of catastrophic losses. Because catastrophe losses can be quite large relative to the resources of the reinsurance industry, the probability of a reinsurance default is significant.[2] Moreover, because there is a lack of transparency in the reinsurance market with respect to reinsurer asset quality, the adequacy of reserve estimates, and the reinsurer's exposure to catastrophic risk, the insurer's and reinsurer's perceived probability distributions of reinsurer default risk are likely to differ. Thus, our model seems to be particularly relevant for analyzing the catastrophe reinsurance market.

Optimal insurance in the presence of default risk has previously been analyzed by Tapiero, et al. (1986) and Doherty and Schlesinger (1990). The former paper considers the effect of default risk on consumers who make an "all or nothing" decision on whether to fully insure. The Doherty-Schlesinger paper is more similar to ours. However, in their model the probability of insurer default is modeled using a Bernoulli distribution rather than a probability density function, as in our analysis. More importantly, in their model the probability of default is known by both the buyer and the seller, i.e., the possibility that informational asymmetries may affect the outcome is not considered. Their main conclusion is that partial insurance is optimal when default is total, while results are indeterminate in the partial-default case even under the restrictive behavioral assumption of constant absolute risk aversion. As a consequence, we focus in this paper on the total-default case since our objective is to investigate the impact of deviant beliefs about the probability distribution of the insurer's total default risk on the design of an optimal insurance contract. This assumption about the indemnity payment is a reasonable representation of the likely payoff in the international reinsurance market. If a reinsurer in Switzerland or Germany, for example, were to default, it is likely to be very difficult for creditors from other countries such as the U.S. to proceed against the reinsurer's remaining assets. The problem would be exacerbated for large catastrophic losses, which could result in a significant short-fall between liabilities and assets. In the individual buyer case, claims payments are likely to be significantly delayed and subject to maximum limits imposed by insurance guaranty funds, so the total-default case may hold as an approximation. This assumption is also seems reasonable if no guaranty fund protection exists.

A theoretical and empirical examination of the market for catastrophe reinsurance has been conducted by Froot (2001). Using a model which does not allow for reinsurer default risk, he shows that optimal reinsurance coverage is full coverage above a deductible, similar to the result from standard insurance theory. However, his analysis of actual coverage patterns in the catastrophe reinsurance market shows that actual reinsurance coverage does not conform to the theory. Instead of reinsuring a high proportion of the largest losses and decreasing proportions of smaller losses, as predicted by theory, actual reinsurance purchasing patterns involve relatively high coverage of smaller events and diminishing marginal coverage, as a function of event size, for larger events.

Froot (2001) offers a number of possible explanations for the difference between theory and practice, mostly involving market imperfections and friction costs. In this paper, we propose an alternative explanation, that is, we seek to determine whether this aggregate profile of reinsurance purchases could be rationalized by the presence of total default risk using the classic expected utility paradigm of insurance economic theory. This provides the second objective of the paper. Specifically, we aim at deriving conditions under which expected utility theory would predict an inverse relationship between the purchase of reinsurance and the size of the industry-wide loss. Under plausible assumptions about buyer risk aversion and the difference between the insurer's and the reinsurer's perceived probability distributions of the reinsurer's default risk, we can generate a coverage pattern similar to that observed in the catastrophe reinsurance market. Thus, market imperfections, other than default risk, may not be needed to explain observed coverage patterns for catastrophe reinsurance.

The remainder of the paper is organized as follows: The model and the assumptions are presented in section 2. Section 3 is devoted to deriving the optimal insurance contract subject to default risk. In section 4, the modeling results are used to propose an explanation for observed coverage patterns in catastrophe reinsurance markets. Section 5 concludes.

2. The model

A risk-averse agent (the buyer) with nonrandom initial wealth w_0 faces a risk of loss denoted by the positive random variable $\tilde{x}$.[3] He has the opportunity to purchase an insurance contract from a risk-neutral insurer in order to reduce his exposure to losses due to the occurrence of random events. The loss $\tilde{x}$ is potentially quite large, exposing the insurer to default risk, i.e., there exists a positive probability that the insurance company will be unable to pay the indemnity in the event of a large realization of $\tilde{x}$. Following the terminology proposed by Johnson and Stulz (1987), contracts subject to default risk are labeled *vulnerable contracts*. The performance of the insurance contract thus depends on the ability of the insurer to make the promised payment.

The probability of total default is assumed to be exogenous in the sense that it is independent of the actions of the insurance buyer. This probability can be interpreted as follows. Let $\tilde{A}r_1(\tilde{x})$

and $\widetilde{L}r_2(\widetilde{x})$ denote the insurer's random assets and random liabilities, respectively. His assets and liabilities are affected by a component which is a deterministic function of the loss, $r_1(x)$ and $r_2(x)$ respectively, and by a component, $\widetilde{A}$ and $\widetilde{L}$ respectively, that is assumed to be independent of the loss. The insurer's assets are assumed to be negatively correlated with the loss, i.e., $r_1 \geq 0$ and $r_1' \leq 0$, and his liabilities are assumed to be positively correlated with the loss i.e., $r_2 \geq 0$ and $r_2' \geq 0$. Therefore, the insurer will be insolvent (solvent) if the amount of his assets is lower (higher) than the amount of his liabilities or, given our terminology, if $a \equiv A/L < (\geq) r_2(x)/r_1(x) \equiv r(x)$, where $r \geq 0$ and $r' \geq 0$ from the assumptions on r_1 and r_2. Hence, r characterizes the insurer's degree of exposure to the loss $\widetilde{x}$. The probability of default conditional on the realization of the loss x is $\text{Prob}[\widetilde{a} < r(x)]$. This conditional probability increases with the loss. Such a characterization of the total default risk implicitly takes into account the correlation between individuals' catastrophic risks; an increase in the individual loss will cause an increase in the aggregate loss in the insurer's portfolio because these losses are positively correlated and therefore this will increase the probability of the insurer's total default risk.[4]

Because of informational asymmetries between the buyer and the insurer with respect to elements such as the quality of the insurer's asset portfolio, the adequacy of its reserves, and its overall exposure to loss, the buyer and the insurer are likely to have different beliefs about the insurer's default risk. That is, they may have different subjective probabilities of default risk.[5] Notice that we do not assume that the buyer has a more specialized knowledge of the loss distribution than the insurer. This implies that our model is not subject to adverse selection. We denote F and f the cumulative distribution function (CDF) and the density function of $\widetilde{a}$

assessed by the buyer, and G and g the CDF and the density function of $\tilde{a}$ assessed by the insurer. The CDF of the catastrophic loss $\tilde{x}$ is denoted Φ, it is defined on the support $[0,\bar{x}]$, where $\bar{x}>0$, and it is assumed to be identically perceived by both parties.[6]

The insurance contract is described by a couple $[I(.),P]$ where P is the premium and $I(x)$ is the indemnity paid by the solvent insurer when the realized loss is x. Under the total-default case, the insured receives no indemnity if the insurer becomes insolvent. The insurance buyer's final wealth is thus

$$(1) \qquad w=\begin{cases} w_0-x+I(x)-P & \text{if } a\geq r(x) \\ w_0-x-P & \text{if } a<r(x). \end{cases}$$

The choice of the theoretical framework in which our problem will be investigated is crucial. Our first idea was to analyze this problem under the assumption of risk aversion to mean-preserving spreads, including the expected utility theory as a special case. Gollier (2000) provides a detailed analysis of optimal insurance design under this framework, and Doherty and Schlesinger (2002) use it to examine the role of securitization in the financing of catastrophe risk. However, such a framework turns out to be much less tractable for our problem than the expected utility theory. In particular, we do not want restrict our analysis to coinsurance contracts, as in Doherty and Schlesinger (2002) in order to highlight the role of the deductible in the optimal insurance policy.[1] As a consequence, we assume that the insurance buyer is strictly risk averse with von Neumann-Morgenstern utility of wealth represented by the twice differentiable function u, with $u'>0$ and $u''<0$, and the (re)insurer is assumed to be risk neutral.

[1] Although insurance contracts containing coinsurance provisions do exist in insurance markets, policies with non-linear payoff features such as deductibles tend to be more common in most property-liability coverages. This includes relatively low risk coverages (in terms of volatility and susceptibility to correlated losses) such as automobile liability insurance as well as commercial coverages such as general liability. Non-linear payoffs are the dominant contractual form in high risk coverages such as catastrophic loss reinsurance and also in securitized risk transfer instruments such as catastrophic risk (CAT) bonds (Cummins, Lalonde, and Phillips 2002).

3. Optimal vulnerable insurance contract

The optimization problem

A feasible insurance contract $[I(.),P]$ is such that the indemnity is non-negative:

(2) $I(x)\geq 0$ for all $x\in[0,\bar{x}]$,

and the premium is determined for any indemnity schedule. The premium P is assumed to be based upon the expected indemnity payment by the risk-neutral insurer:

(3) $$P=c\left[\int_0^{\bar{x}} I(x)\overline{G}(r(x))d\Phi(x)\right],$$

where $\overline{G}(r)=1-G(r)$ denotes the insurer's probability of solvency assessed by the insurer itself and $c(.)$ is the cost function satisfying $c(0)=0$ and $c'(e)\geq 1$ for all $e > 0$.[7] Notice that, in general, the premium is not actuarially fair. In the following discussion, the premium is not actuarially fair unless specifically indicated.

The first-best vulnerable insurance contract is the couple $[I(.),P]$ that maximizes the insurance buyer's expected utility of final wealth subject to the above-mentioned constraints. This problem can be written as

(4) $$\underset{I(.),\,P}{Max}\int_0^{\bar{x}}\left\{u(w_0-x+I(x)-P)\overline{F}(r(x))+\int_0^{r(x)}u(w_0-x-P)\,dF(a)\right\}d\Phi(x)$$

subject to conditions (2) and (3),

where $\overline{F}(r)=1-F(r)$ is the insurer's probability of solvency assessed by the buyer.

Optimal insurance design

The next proposition characterizes the design of an optimal insurance contract when the insurer's default risk is perceived differently by the buyer and insurer. We define

$\delta_b(r(x)) = f(r(x))/\overline{F}(r(x))$ and $\delta_s(r(x)) = g(r(x))/\overline{G}(r(x))$ as the *default rates* (also known as *hazard rates*) of the buyer and of the insurer, respectively. For infinitesimal value $h > 0$, $\delta_i(r(x))h = \mathrm{Prob}_i\left[r(x) < \tilde{a} \leq r(x) + h \mid \tilde{a} > r(x)\right]$ for $i = b, s$ is the probability of default at $r(x) + h$ given that the insurer is solvent at $r(x)$, as it is estimated by the buyer (b) or the insurer (seller = s). If $\delta_i(r(x))$ becomes small (large), immediate default is less (more) likely to occur and the distribution of $\tilde{a}$ is heavy-tailed (light-tailed).

Proposition 1. *Suppose that the insurance contract is subject to default risk which is perceived differently by the risk-averse buyer and by the risk-neutral insurer. The optimal insurance contract* I^**, the solution of the maximization problem (4), takes the following form. If the following inequality holds for all* $x \in [0, \overline{x}]$ *and* w*:*

(5) $$A_u(w) \geq r'(x)[\delta_b(r(x)) - \delta_s(r(x))],$$

then a deductible $D \in [0, \overline{x}]$ *exists such that:*

(6) $$I^*(x)\begin{cases} > 0 & \text{if } x > D \\ = 0 & \text{otherwise.} \end{cases}$$

The marginal coverage satisfies:

(7) $$I^{*\prime}(x) = 1 - T_u(w)r'(x)[\delta_b(r(x)) - \delta_s(r(x))] \text{ for all } x : I^*(x) > 0,$$

where $A_u \equiv -u''/u'$ *is the degree of absolute risk aversion of the buyer,*

$T_u = A_u^{-1}$ *is his index of absolute tolerance towards risk,*

$w = w_0 - x + I^*(x) - P$,

$\delta_b(r(x)) = f(r(x))/\overline{F}(r(x))$*, and*

$\delta_s(r(x)) = g(r(x))/\overline{G}(r(x))$

The functions $\delta_b(r(x))$ *and* $\delta_s(r(x))$ *are the default rates (hazard rates) associated with the subjective probability of performance estimated by the buyer and the insurer, respectively.*

The proof of this proposition is shown in the Appendix. The design of an optimal vulnerable insurance contract is determined by the buyer's risk aversion and the default rates perceived by the buyer and the insurer. If the buyer is sufficiently risk averse so that condition (5) is satisfied, then the optimal vulnerable insurance policy contains a deductible $D \geq 0$ above which indemnity payments are made. The critical level for the buyer's index of absolute risk aversion is the product of the insurer's marginal exposure to risk of loss and of the difference between the buyer's and the insurer's subjective default rates. It is noteworthy that if the insurer's subjective default rate is not less than the buyer's, i.e., if $\delta_s(r) \geq \delta_b(r)$ for all r, then condition (5) holds whatever the attitude towards risk of the risk-averse buyer. If the buyer is not sufficiently risk averse, i.e., condition (5) is violated for some realized losses, then the indemnity schedule can take basically any form.

When the indemnity is positive, the optimal marginal coverage expressed in equation (7) is equal to unity minus a specific term. This term is the product of the buyer's index of absolute risk tolerance, of the insurer's marginal exposure to catastrophic loss, and of the difference between the buyer's and the insurer's subjective default rates. Under condition (5), the optimal indemnity schedule increases with the loss above the deductible D. The marginal coverage is lower or higher than unity depending on whether the buyer's perceived default rate is higher or lower than the insurer's perceived default rate. The divergent perception of the insurer's default risk by the buyer and the insurer, which leads to different subjective default rates, entails that the optimal marginal coverage differs from unity. The magnitude of the deviation from unity of the optimal marginal coverage is directly related to the buyer's risk tolerance, the difference between the buyer's and the insurer's perceived default rates, and the insurer's exposure to the loss x.

Divergent beliefs about the insurer's default risk entails that the optimal marginal coverage above the deductible differs from the optimal one in the standard default-free model, which is equal to unity. The more risk averse the buyer, the less sensitive to the divergence in the perception of default risk the marginal coverage is.

When both parties have the same perception about the insurer's default risk, i.e. $F \equiv G$, condition (5) is trivially satisfied for a risk-averse buyer and, from equation (7), the optimal marginal coverage above the deductible is equal to one, i.e. $I^{*}'(x)=1$ for all $x > D$. The following corollary is thus derived as a particular case of Proposition 1.

Corollary 1. *If the risk-averse buyer and the risk-neutral insurer have the same perception about the insurer's default risk, the optimal insurance contract displays full insurance above a deductible: there exists* $D \in [0, \bar{x}]$ *such that* $I^{*}(x) = \max[x - D, 0]$.

The optimality of full insurance above a deductible, shown by Arrow (1963) under the implicit assumption that the insurance policies are performing contracts, also holds in the presence of an exogenous risk of default, if the perception of this risk is identical for the two parties. It should be noticed that the *form* of the optimal reinsurance policy is independent of the exogenous partial payoff.

Optimal deductible

The following proposition characterizes the optimal deductible in the total-default case.

Proposition 2. *Consider the same economic environment as in Proposition 1.* Suppose *that condition (5) holds and that the insurance contract is considered as actuarially fair by the insurer, i.e.,* $c'(e)=1$ *for all* e. *If*

(8) $\delta_b(r(x)) \geq \delta_s(r(x))$ *for all* $x \in [0, \bar{x}]$ *and* $EF(r(\tilde{x})) \geq EG(r(\tilde{x}))$,

then the optimal deductible is positive under a total default risk.

The proof of this proposition is shown in the Appendix. If both parties have the same perception of the reinsurer's default risk, condition (8) holds trivially and the following corollary is derived from the above proposition.

Corollary 2. *Suppose that the risk-averse buyer and the risk-neutral insurer have the same perception about the insurer's default risk. Under an actuarially fair insurance contract, the optimal deductible is positive if the default is total.*

When the two parties agree on the insurer's default risk, the optimal deductible is positive under a risk of total default, even if the insurance contract is sold at an actuarially fair price. The existence of a risk of total default thus generates another exception to the standard result that full insurance is optimal when the premium is actuarially fair (Arrow 1963).

The existence of a divergence in the perception of the insurer's default risk makes the results less straightforward, as presented in Proposition 2. Under a risk of total default, the optimal deductible is positive when the insurance contract is considered as fair by the insurer and under the additional assumption (8), which means that the buyer is, in some sense, more pessimistic than the insurer. That is, the buyer's subjective rate of total default and his subjective probability of total default is larger than the insurer's perceptions of these two quantities.

The optimal deductible of an actuarially fair vulnerable insurance contract, from the buyer's viewpoint, is the result of two effects. The first is due to the existence of a total default risk. As shown in Corollary 2, when both parties have the same perception of default risk, it induces the buyer to select a positive deductible. The second effect is caused by the divergence in the perception of the insurer's total default risk. If the buyer is more pessimistic than the insurer, as defined by condition (8), then this divergence creates a kind of transaction cost. The buyer considers that the insurance premium is actuarially unfair and therefore he is induced to select a positive deductible. In this case, both effects go in the same direction and lead to Proposition 2. On the contrary, if the buyer is less pessimistic than the insurer, in the sense that the two inequalities in condition (8) are reversed, then the buyer believes that the insurance premium is smaller than the actuarially fair premium and therefore is induced to reduce his deductible. The total effect is thus indeterminate.

4. Catastrophe reinsurance with vulnerable contracts

The traditional hedge for the primary insurer is reinsurance.[8] Reinsurers should be able to bear catastrophe risk that is undiversifiable to primary insurers by spreading this risk.[9] Nevertheless, recent studies of the market for catastrophic risk suggest that primary insurance companies retain large exposure to catastrophe events (e.g., Froot 2001). Several explanations for the paucity of catastrophic risk sharing have been proposed. Most of them focus on the supply side: insufficient reinsurance capital, reinsurers' market power, high frictional costs of reinsurance, information asymmetries, and other market imperfections. On the demand side, this paucity is often explained by the presence of regulatory constraints, the existence of governmental funds for disaster assistance, or some behavioral factors that do not seem to be easily justified with an utility-based approach.

The aggregate profile of reinsurance purchases is presented in Figure 1. It shows the fraction of catastrophe losses that insurers protect through reinsurance, averaged across insurers.[10] This fraction of protection is high for relatively small catastrophe losses, but low for moderate and large catastrophe events. Marginal exposure reinsured by primary insurers first increases for insured losses lower than $500 million, and then declines with the size of the event, falling to a level of less than 25% for catastrophe events generating a $6 billion industry loss. This profile can be viewed as the reinsurance profile of a *representative* primary insurer of the insurance industry. Figure 1 can thus be reinterpreted as the fraction of the representative insurer losses covered through its own reinsurance contract.[1]

[INSERT FIGURE 1 HERE]

In this section, we examine whether this aggregate profile of reinsurance purchases could be rationalized by the presence of default risk using the classic expected utility paradigm of insurance economic theory. Specifically, we aim at deriving conditions under which the utility theory would predict an inverse relationship between the purchase of reinsurance and the size of the representative insurer losses.

If the primary insurer and the reinsurer have the same perception about the reinsurer's default risk, Corollary 1 shows that the optimal insurance policy displays full insurance above a deductible. As a consequence, the retention of a large fraction of catastrophic losses by the primary insurers cannot be explained by the presence of default risk alone.

By contrast, if the insurer and reinsurer have deviant beliefs about the reinsurer's default risk, the optimal marginal coverage expressed in equation (7) can be rewritten as

(9) $$I^{*}{}'(x) = 1 - T_u(w) r'(x) \Delta(r(x)),$$

for all $x \in [D, \bar{x}]$, where $\Delta(r) = \delta_b(r) - \delta_s(r)$ is the difference of the subjective default ratios. Under condition (5), the optimal vulnerable reinsurance contract displays coinsurance above a deductible D if $\Delta(r(x)) \geq 0$ for all x. The second derivative of the optimal coverage is

(10) $$I^{*}{}''(x) = T_u(w)[r'(x)\,\Delta(r(x))]^2 \left\{ T_u'(w) - \frac{\partial[r'(x)\,\Delta(r(x))]/\partial x}{[r'(x)\,\Delta(r(x))]^2} \right\},$$

for all $x \in [D, \bar{x}]$.

Because the first two factors on the right hand side (RHS) of equation (10) are positive, the shape of the first-best indemnity schedule depends on the comparison of the two RHS terms in brackets. The term T_u' is the derivative of the primary insurer's index of absolute tolerance towards risk. It is non-negative under the widely accepted assumption of non-increasing absolute risk aversion (Pratt 1964). The ratio is related to the derivative of the reinsurer's marginal exposure to catastrophic risk multiplied by the difference between the subjective default rates, with respect to the loss. The optimal coverage is concave if

(11) $$T_u'(w) \leq \frac{\partial[r'(x)\,\Delta(r(x))]/\partial x}{[r'(x)\,\Delta(r(x))]^2} = \frac{\Delta'(r(x))}{\Delta^2(r(x))} + \frac{r''(x)}{[r'(x)]^2}\frac{1}{\Delta(r(x))},$$

for all w and $x \in [D, \bar{x}]$, where $\Delta'(r) = \partial\Delta(r)/\partial r$. A necessary condition for the concavity of the optimal indemnity function is that $\Delta(r)$ increases with r or $r(x)$ is convex with x.

If the representative primary insurer's utility exhibits hyperbolic absolute risk aversion, then the index of absolute tolerance towards risk is linear in wealth, i.e., $T_u(w) = a + dw$ where $d \geq 0$ under non-increasing absolute risk aversion. When $a = 0$, u exhibits constant relative risk aversion (CRRA) and the index of relative risk aversion $1/d$ is usually suggested to be between 1 and 4 (Drèze 1987). It is worthwhile to notice that the marginal index of absolute tolerance towards risk can be rewritten as

(12) $$T_u'(w) = -1 + T_u(w) P_u(w)$$

where $P_u \equiv -u'''/u''$ is the index of absolute prudence (Kimball 1990). This entails that T_u' is positive and close to zero if P_u is not too much higher than A_u, i.e., the primary insurer's prudence is not too much higher than his risk aversion.

The RHS part of inequality (11) is thus positive if the primary insurer's perceived marginal default rate is higher than the reinsurer's marginal default rate and if the degree of the reinsurer's exposure to catastrophic risk is a non-concave function of the loss. Hence, the optimal coverage is increasing and concave with losses above a deductible if, from condition (5), the primary insurer is sufficiently risk averse and, from condition (11), his index of absolute risk aversion does not increase "too fast" with wealth. These two conditions hold if the primary insurer's utility function is CARA with an index of absolute risk aversion sufficiently high.

The optimal indemnity schedule $I^*(x)$ is illustrated in Figure 2 under the behavioral conditions (5) and (11), total default risk and actuarially fair reinsurance contract. A critical catastrophe level $x_0 \in [D, \bar{x}]$ exists such that the optimal indemnity function as a fraction of losses, denoted $I^*(x)/x$, increases in the interval $[D, x_0]$ and decreases in the interval $[x_0, \bar{x}]$.

The shape of $I^*(x)/x$ looks like the curve generated by the relationship between the fraction of pooled insurer exposure covered by reinsurance and the size of industry-wide events presented in Figure 1. Therefore, the large retention of catastrophic risk by primary insurers may be partly explained by divergent beliefs about the reinsurer's risk of default.

[INSERT FIGURE 2 HERE]

If $\Delta(r)$ decreases with r and $r(x)$ is concave with x, then the ratio in brackets in equation (11) is negative and, therefore, the optimal coverage is convex in the catastrophe loss. As a consequence, the reinsurance coverage as a fraction of exposure should increase with the size of the event.

A second point to take from Figure 1 is that the rate of reinsurance for insured losses higher than $1.5 billion increases over time. For instance, for events of $7 billion, this rate is roughly equal to 8% in 1970, 12% in 1980, 17% in 1990, and 20% in 1994. Such a trend can be justified with standard explanations, such as an increase in supply of reinsurance and/or a reduction of the reinsurers' market power inducing them to increase their capital. An alternative explanation could also be provided through our model. Since the primary insurer and the reinsurer have subjective default rates, their perceptions about the reinsurer's risk of default could evolve over time. For instance, the occurrence of natural disasters provides information about the ability of the reinsurance company to make the promised payment in the event of a large catastrophe. This may entail a convergence in the perception of default risk by the two parties. The difference in default rates $\Delta(r)$ may thus decrease for all r, and especially for high values of r. From equation (9), this implies that the optimal marginal coverage above the deductible should increase over time. At the limit, if both parties have the same perception about default risk, then full insurance above the deductible would be optimal, as shown in Corollary 2.

In this section, we have shown that there are mathematical conditions under which the optimal insurance indemnity schedule is increasing and concave as in Figure 2. We now consider the issue of whether these mathematical conditions are likely to be observed in practice. As mentioned above, the term T_u' will be non-negative under the widely-accepted assumption of non-increasing absolute risk aversion. Also as mentioned, under hyperbolic absolute risk aversion, T_u' is equal to the reciprocal of the index of relative risk aversion, which would place its value in the range from 0.25 and 1.0. The question is whether the right hand side of expression (11) is likely to exceed that T_u'.

To provide a rough estimate of the possible magnitude of the right hand side of expression (11), we first need an assumption about $r(x)$. To provide the basis for discussion, we adopt the very simple assumption that $r_1(x)=1$ and $r_2(x)=1+x$, where x is the catastrophe loss, expressed as a proportion of the insurer's non-catastrophe losses.[11] This provides a model of the insurer's asset position in the instant following a catastrophe, i.e., the loss has resulted in an increase in the insurer's liabilities but has not yet resulted in any payments to policyholders. In this simple case, $r(x)=r_1(x)/r_2(x)=1+x$, which satisfies the required conditions that $r \geq 0$ and $r' \geq 0$. In fact, in this case $r''=0$, and the second term on the right hand side of (11) vanishes.

Next we consider the likely behavior of $\Delta(r)=\delta_b(r)-\delta_s(r)$. In order for condition (11) to hold under our assumptions, we require that $\Delta'(r)>0$. To provide some information on this quantity, we consider empirical evidence on hurricane and earthquake loss distributions presented in Cummins, Lewis, and Phillips (CLP) (1999). CLP fit probability density functions to actual historical U.S. catastrophic hurricane and earthquake losses covering the period 1949-

1997, where losses are adjusted for price level changes as well as for changes in the amount of property insured for loss. Specifically, we used the distributions fitted by CLP to Property Claims Services (PCS) data on property catastrophes expressed in 1994 dollars and adjusted for the level of exposures using U.S. Census Bureau data on population (the *population-adjusted* estimates) and, alternatively, on U.S. Census data on the value of owner-occupied housing (the *housing-value adjusted* estimates).[12] CLP find that the lognormal and Burr 12 distributions provide excellent models for the adjusted property catastrophe losses.

To consider the behavior of $\Delta'(r)$, we use the parameter estimates from CLP for the lognormal and Burr 12 distributions. The analysis is conducted once using the lognormal distributions fitted to the population-adjusted and housing-value adjusted catastrophe losses and then again using the Burr 12 with the two sets of catastrophe losses. We assume that the insurer is more pessimistic than the reinsurer about the reinsurer's solvency and adopt the riskier of the estimated population-adjusted and housing-value adjusted lognormal (Burr 12) distributions to represent the insurer's perception of the reinsurer's insolvency risk. We convert the uncertainty about the distribution of catastrophe losses into uncertainty about net assets by modeling net assets as: $A_0 - L_0(1+\tilde{x})$, where A_0 and L_0 are non-stochastic starting values for assets and liabilities and $\tilde{x}$ is a random variable representing catastrophe losses.[13] For both the lognormal and the Burr 12 cases, the calculations show that $\Delta'(r) > 0$ and $\Delta'(r(x)) / [\Delta(r(x))]^2 \in$ [1,50). These exploratory results imply that it is possible for condition (11) to be satisfied based on realistic assumptions about risk aversion and the distribution of catastrophic property losses. Thus, it may not be necessary to rely on factors such as costly external capital, market power of reinsurers, and transactions costs to explain the increasing and concave indemnity schedule for actual reinsurance purchases shown in Figure 1.

5. Conclusions

This paper extends the classic expected utility theory model of optimal insurance contracting to consider the case where the insurer's total default probability is non-zero and the buyer and insurer have divergent beliefs about the probability distribution of total default risk. The backdrop for our analysis are two principal results from the economic theory of insurance: (1) Risk-averse buyers will fully insure at actuarially fair prices, and (2) if insurance is priced at higher than actuarial rates, optimal insurance involves full insurance above a deductible (Arrow 1963, Mossin 1968). The latter result also implies that the first derivative of the optimal indemnity schedule (the marginal indemnity) with respect to the loss is equal to unity above the deductible. We seek to determine how these results are affected by non-zero insurer total default probabilities and by divergent beliefs about these probabilities.

In summarizing our results, we assume in all cases that default risk is non-zero and that the buyer meets our criterion for sufficient risk aversion (condition (5)). We first summarize our results based on the assumption that prices are not necessarily actuarially fair: (1) if the insurance buyer is sufficiently risk averse, then the optimal insurance policy contains a deductible above which payments are made. (2) If the buyer and the insurer have the same perception of insurer default risk, then the derivative of the indemnity schedule above the deductible is equal to unity, as in the traditional case. Thus, default risk alone does not change the form of the contract derived by Arrow (1963) for the default-free case. (3) If the parties to the contract have divergent beliefs about insurer default risk, then the optimal marginal indemnity above the deductible is greater or smaller than one, depending on whether the buyer's assessment of default risk is more optimistic or pessimistic than the insurer's. If the buyer is more pessimistic than the insurer, which may be more likely in practice, the optimal insurance contract involves coinsurance above a deductible. Thus, divergent beliefs about default risk, do change the form of the optimal policy in comparison with the traditional case.

One of our principal results is based on the assumption that prices are actuarially fair. This result also requires that the buyer either has the same or more pessimistic beliefs about insolvency risk than the insurer: (4) If insurance is actuarially fair and the buyer receives nothing in the event of default, full coverage above a positive deductible is optimal. The risk of total default and divergent assessments of default probabilities create a type of transactions cost that leads to coverage patterns analogous to what would be observed in a market with default free insurance sold at actuarially unfair prices.

As an application of our model, we analyze the market for catastrophe reinsurance. We seek to explain the empirical relationship between the fraction of pooled insurer exposure covered by reinsurance and the size of the industry-wide events highlighted by Froot (2001). If the representative primary insurer's risk aversion is sufficiently high and not too sensitive to changes in wealth, then the optimal indemnity schedule could be increasing and concave above the deductible. Such a shape could contribute to rationalizing the representative insurers' low reinsurance protection at high layers of exposure. Convergence in the perception of the default risk over time may help to explain the increase in reinsurance protection over time that is noticed in Froot's (2001) empirical analysis. The parameter values necessary to generate the observed indemnity schedule are shown to be feasible in terms of reasonable assumptions about risk aversion and actual property catastrophe loss distributions.

The discussion on this paper also emphasizes the importance of securitization as a mechanism for financing catastrophic losses. A $100 billion earthquake loss, for example, would create an insolvency problem for numerous insurers and reinsurers but would represent less one percent of the value of U.S. stocks and bonds (Cummins, Doherty and Lo 2002). Moreover, shifting these large loss events from insurance markets to financial markets would reduce the probability of reinsurer failure and hence mitigate the market inefficiencies resulting from non-zero bankruptcy probabilities. There are a number of problems to be worked out before the CAT securities market can develop into an efficient and liquid mechanism for financing CAT risk. However, it seems clear that securitization offers the ultimate solution to the CAT loss financing problem.[14]

Appendix

Proof of Proposition 1

For a given premium P, problem (4) is solved by using Kuhn-Tucker conditions for $I(x)$ for all $x \in [0, \bar{x}]$ because the marginal indemnity $I'(x)$ does appear neither in the objective function nor in the constraints. The first-order condition is

(A1) $\quad \bar{F}(r(x))u'(w_0 - x + I(x) - P) + \lambda(x) - \mu\bar{G}(r(x))c' = 0$ for all $x \in [0, \bar{x}]$

where c' is evaluated at $\left[\int_0^{\bar{x}} I(x)\bar{G}(r(x))d\Phi(x)\right]$, μ and $\lambda(x)$ are the Lagrangian multipliers associated respectively to constraints (3) and (2), with

(A2) $\quad \lambda(x)\begin{cases} = 0 & \text{if } I(x) > 0 \\ \geq 0 & \text{otherwise.} \end{cases}$

Assume that the probability of performance is always strictly positive from the insurer's viewpoint, i.e., $\bar{G}(r) > 0$ for all r. Equation (A1) can thus be rewritten as

(A3) $\quad \bar{G}(r(x))\{h(r(x))u'(w_0 - x + I(x) - P) + \theta(x) - \mu c'\} = 0$ for all $x \in [0, \bar{x}]$,

where $h(r) = \bar{F}(r)/\bar{G}(r)$ and $\theta(x) = \lambda(x)/\bar{G}(r(x))$.

For all $x : I(x) = 0$, we deduce from equation (A3) that

(A4) $\quad K(x) \equiv \bar{G}(r(x))\{h(r(x))u'(w_0 - x - P) - \mu c'\} \leq 0$.

Its first derivative is

(A5) $$\begin{aligned} K'(x) &= -r'(x)g(r(x))\{h(r(x))u'(w_0 - x - P) - \mu c'\} \\ &+ \bar{G}(r(x))\{r'(x)h'(r(x))u'(w_0 - x - P) - h(r(x))u''(w_0 - x - P)\}. \end{aligned}$$

The first RHS term is positive because of (A4) and $r' \geq 0$. Noting that $h'(r)/h(r) = [g(r)/\bar{G}(r) - f(r)/\bar{F}(r)] = [\delta_s(r) - \delta_b(r)]$, the second RHS term is positive if

(A6) $A_u(w) \geq r'(x)[\delta_b(r(x)) - \delta_s(r(x))]$ for all $x \in [0, \bar{x}]$ and w.

Therefore, a deductible $D \in [0, \bar{x}]$ exists such that $K(x)(x - D) \geq 0$ for all $x \in [0, \bar{x}]$. As a result, the optimal coverage function takes the form (6) under condition (5).

For all $x : I(x) > 0$, equation (A3) can be rewritten as

(A7) $h(r(x))u'(w_0 - x + I(x) - P) - \mu c' = 0$.

Differentiating equation (A7) with respect to x and rearranging the terms lead to the marginal coverage expressed in equation (7).

Proof of Proposition 2

The maximization of problem (4) with respect to P yields:

(A8) $\mu = E[u'(\tilde{w}_1)\overline{F}(r(\tilde{x}))] + E[F(r(\tilde{x}))u'(\tilde{w}_2)]$,

with $w_1 = w_0 - x + I^*(x) - P$, $w_2 = w_0 - x - P$ and c' is evaluated at $\left[\int_0^{\bar{x}} I(x)\overline{G}(r(x))d\Phi(x)\right]$.

Equation (A1) can be rewritten as

(A9) $\lambda(x) = -\overline{F}(r(x))u'(w_1) + \mu\overline{G}(r(x))c'$ for all $x \in [0, \bar{x}]$.

Taking the expectation of the above equality with respect to $\tilde{x}$ and replacing μ by its expression in equation (A8) yield

(A10) $E\lambda(\tilde{x}) = -E[u'(\tilde{w}_1)\overline{F}(r(\tilde{x}))] + c'E\overline{G}(r(\tilde{x}))\{E[u'(\tilde{w}_1)\overline{F}(r(\tilde{x}))] + E[u'(\tilde{w}_2)F(r(\tilde{x}))]\}$.

It can be rewritten as

(A11)
$$\begin{aligned} E\lambda(\tilde{x}) = {} & Eu'(\tilde{w}_1)E\overline{F}(r(\tilde{x}))(c'-1) + c'E\overline{G}(r(\tilde{x}))E\{[u'(\tilde{w}_2) - u'(\tilde{w}_1)]F(r(\tilde{x}))\} - \mathrm{cov}[u'(\tilde{w}_1), \overline{F}(r(\tilde{x}))] \\ & + c'Eu'(\tilde{w}_1)[E\overline{G}(r(\tilde{x})) - E\overline{F}(r(\tilde{x}))], \end{aligned}$$

where cov is the covariance operator. The first right-hand-side (RHS) term is positive if the insurance is costly, i.e. $c' > 1$, and it is equal to zero under fair insurance, i.e. $c' \equiv 1$. The second RHS term is positive because $w_1 > w_2$ and u is concave. Since $u'' < 0$ and $\overline{F}(r(x))$ decreases with x, the covariance term is negative if $I^{*\prime}(x) \leq 1$ for all $x : I^*(x) > 0$. Under Proposition 1, this entails that $\delta_b(r(x)) \geq \delta_s(r(x))$ for all $x \in [0, \overline{x}]$. The last RHS term is non-negative if $EG(r(\tilde{x})) \leq EF(r(\tilde{x}))$. Consequently, under fair insurance, $E\lambda(\tilde{x}) > 0$. This implies that the non-negativity constraint (2) must be binding for some x with a positive probability. This means that the optimal deductible is positive.

Notes

1. For a review of the literature, see Schlesinger (2000).

2. Past decades have shown a dramatic increase in catastrophic losses due to natural disasters. Seven of the ten most costly insured property losses have occurred since 1990. Hurricane Andrew, the largest U.S. catastrophe on record, cost insurers $18.6 billion in 1992. The second largest catastrophe is the Northridge earthquake of 1994, with insured losses of $13.8 billion, and the third most costly event was Typhoon Mireille in Japan in 1991 ($6.7 billion). It has become widely accepted that a single hurricane or earthquake could generate losses of $100 billion or more (Applied Insurance Research, 1998). The global reinsurance industry has $125 billion in equity capital (Cummins and Weiss 2000).

3. Belief deviance is also an expected result of the cost of sharing information (Marshall 1991).

4. Doherty and Dionne (1993) examine the financing of catastrophic risk using a decomposition of the insured losses into systemic (i.e., undiversifiable) and idiosyncratic (i.e., diversifiable) parts. They show that a decomposed risk transfer that efficiently uses this decomposition to apply the mutuality principle and the risk transfer principle weakly dominates a simple transfer in which the insured loss is not decomposed. Doherty and Schlesinger (2002) reconsider this problem and derive a variable participation insurance policy defined as a convex mixture of a non-participating contract (with a fixed premium) and a fully participating contract (with a variable premium) in which the level of participation is endogenous.

5. Belief deviance is also an expected result of the cost of sharing information (Marshall 1991).

6. An alternative approach would have been to assume that both parties have the same CDF of the insurer's ratio $\tilde{a}$ but have different CDFs of the catastrophe loss $\tilde{x}$. If G is the CDF of $\tilde{a}$, Φ and Ψ are the CDF of $\tilde{x}$ for the insurer and the buyer, respectively, then the buyer's subjective survival function can be rewritten as $1 - F(r(x)) = [1 - G(r(x))]\Psi'(x)/\Phi'(x)$. This approach leads to a similar optimization problem.

7. In a default-free model, assuming that the reinsurer is risk neutral and that the premium depends only on the expected indemnity, it is well known that the optimal reinsurance contract provides full coverage above a straight deductible (Arrow 1963). Therefore, these assumptions in our model will allow us to compare easily the impact of default risk on the optimal reinsurance policy.

8. In our model, primary insurers are considered to behave as if they were risk averse. The (apparent) risk aversion of the firm may be due to market imperfections (Greenwald and Stiglitz 1993), to the direct and indirect costs of financial distress, to the existence of taxes that are a convex function of earnings (Smith and Stulz 1985), or to information asymmetries that raise the cost of external funds relative to internal funds (e.g., Smith and Stulz 1985, Froot, Scharfstein and Stein 1993).

9. Reinsurers are treated as risk neutral in our model. Because they can diversify globally, we argue that reinsurers will be closer to risk neutrality than primary insurers. However, because we primarily analyze the decision making of the primary insurer, the assumption of reinsurer risk neutrality is not critical to our results except when we consider cases where insurance prices are actuarially fair. A risk averse reinsurer would not issue policies at actuarially fair prices.

10. Specifically, the graph shows the amount of a marginal dollar of industry-wide loss that is reinsured against catastrophes in a sample of insurance companies that purchase reinsurance through Guy Carpenter & Company, the world's largest reinsurance broker.

11. The approach of converting the loss distribution into a *damage degree distribution* is commonly used in the actuarial literature for studying loss severity distributions (e.g., Klugman, Panjer, and Willmot 1998).

12. Property Claims Services is an insurance industry statistical agent. The distributions were shifted so that the support in each case is $x \in (1, \infty)$. The parameter estimates are provided in CLP, Table 3.3.

13. The catastrophe loss distributions are used in this approach to introduce divergent opinions about the reinsurer's insolvency risk based on real-world estimates of catastrophe losses. This approach is expected to be more realistic than using hypothetical distributions for assets and liabilities that might bear little resemblance to actual risks in the catastrophe reinsurance market.

14. For further information on securitization, see Doherty (1997) and Jaffee and Russell (1997).

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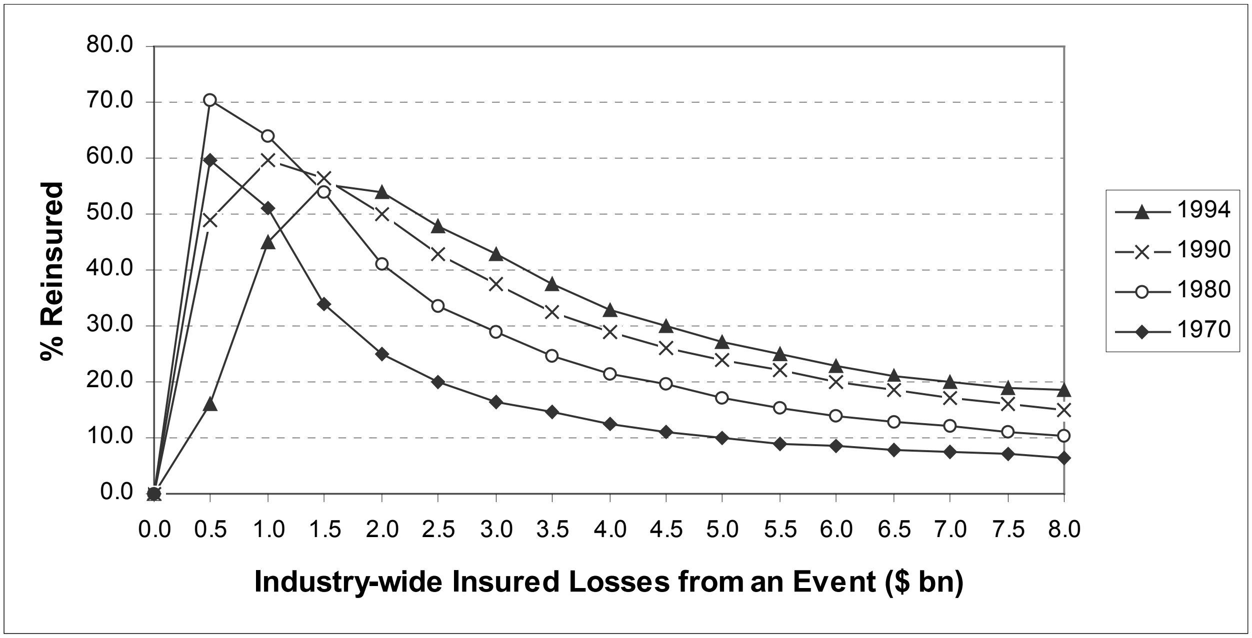

Figure 1. Percentage of Exposure that Insurance Companies Reinsure, by various event sizes, (Froot 2001, Figure 2).

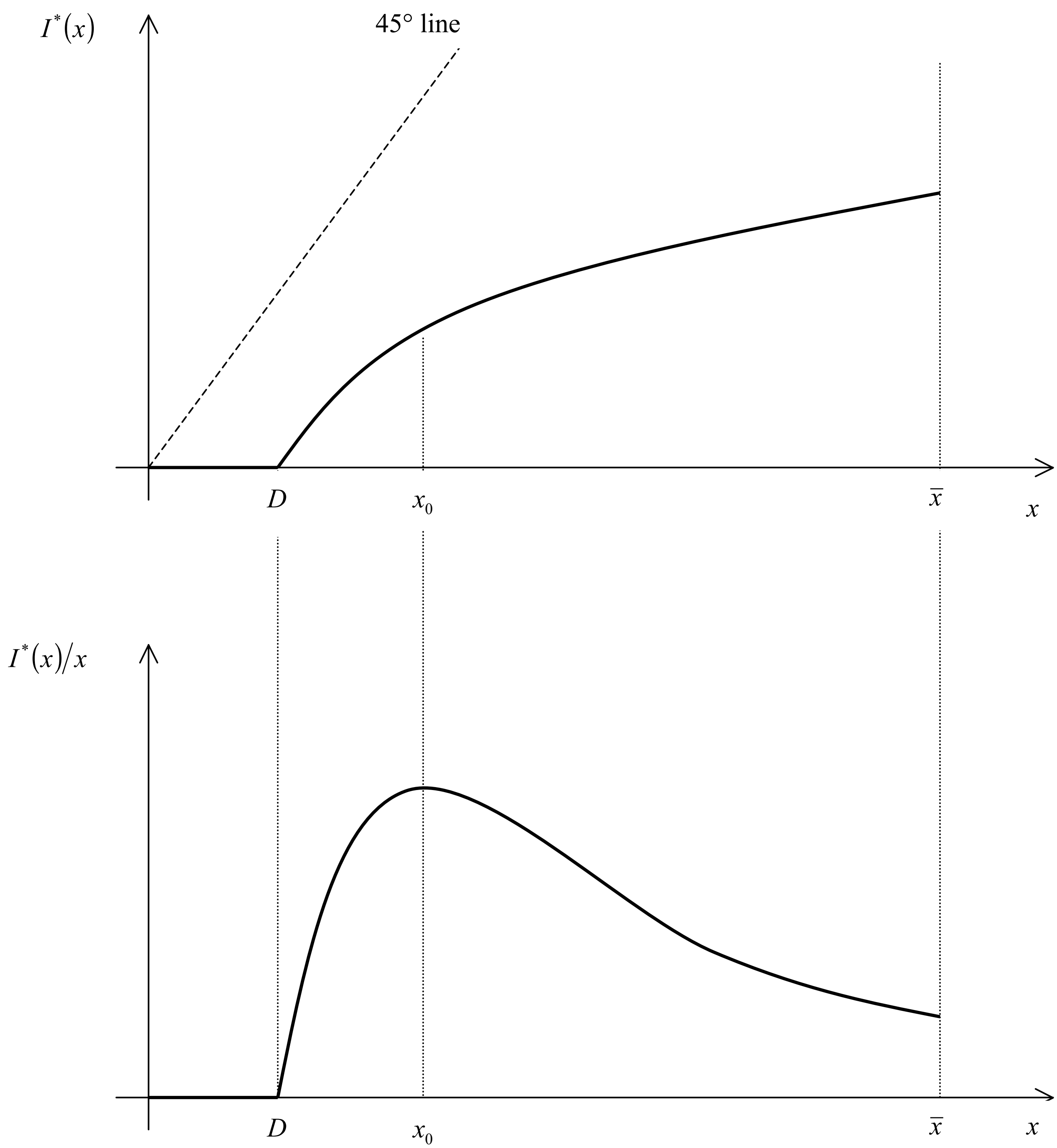

Figure 2. Optimal fair insurance coverage subject to a total default risk, in value and in fraction of losses.

[1]The way the data on marginal coverages in Figure 1 were derived permits us to interpret the figure as distinguishing the coverage pattern of an average or representative firm. The figures were derived by linking "individual firm retention and exhaustion loss amounts to a level of industry-wide losses. This is done using data on US regional market shares for each firm and years from A.M. Best" (Froot 2001, pp. 535-536). Thus, a firm with 10 percent market share is calculated to incur 10 percent of industry wide losses by region so that a reinsurance "layer of $100 million (limit) in excess of $150 million (retention) is calculated to provide protection for industry-wide losses of between $1.5 billion and $2.5 billion" (Froot 2001, p. 536). The estimated amounts this firm would collect from reinsurance from an event of this magnitude were obtained using the parameters of its own actual reinsurance contracts. The horizontal axis in Figure 1 is expressed in terms of industry-wide losses in order to provide a common basis for comparison of coverages across insurers. For further discussion, see Froot and O'Connell (1997).