

On the Role of Good Faith in Insurance Contracting

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Abstract

The law of insurance contracts provides that if the policyholder is found to have knowingly misrepresented material facts about his risks in his application, the insurer can refuse to pay any claims. We examine how this mitigates adverse selection. We extend the Rothschild-Stiglitz (1976) model of competitive insurance markets to the situation where individuals receive a signal of their risk type, and costly verification of the individuals' risk type and/or signal is possible. We show that the insurer's optimal investigation strategy is probabilistic, and contingent on the findings, rewards for truth-telling and penalties for misrepresentation are tailored to achieve better separation of types. The good faith principle enlarges the parameter set for which equilibrium exists, and yields a Pareto improvement. We also examine how the burden of proof should be optimally placed on the insurer or the insured depending on the two parties' costs of proving good or bad faith.

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1 Introduction

The law of insurance contracts usually stipulates that an applicant for insurance must not deliberately conceal any relevant information about his risks from the insurer. This principle of good faith (*uberrima fides*) is implemented with different strictness under common law or statute law. In the United States and the United Kingdom, at the time of making the contract, the insured is obliged to disclose to the insurer all material information affecting the risk; see Clark (1998, pp. 583).¹ The insurer can cancel the contract *ex post facto* and refuse to pay any claims if wilful misrepresentation is found. In France, courts may order a lower payout on the claim even if the misrepresentation is not intentional; if it is, the insurer gets no indemnity and may be subject to an additional fine (Farshian, 2002; Lambert-Faivre, 1991).

The good faith principle can mitigate adverse selection at a low cost, since it allows the insurer to verify the insured's assertions *ex post facto*, only if a claim is filed. Dixit (2000) examines this in an extension of the Rothschild-Stiglitz (1976) model (henceforth R-S) of a competitive insurance market. A random investigation will be carried out when a customer who has asserted himself to be low-risk files a claim. If he is found to be truthful, the insurance indemnity paid exceeds his loss; if untruthful, the claim is denied. This combination of reward and punishment enables better separation of types, and achieves a Pareto improvement over the R-S equilibrium. Lastly, the good faith principle extends the range of high risk and low risk proportions for which a competitive equilibrium exists.

In Dixit's model, individuals are perfectly privately informed about their risk types, so any misrepresentation is intentional. In reality, people may be unaware of their risk type and believe themselves to be low-risk in good faith. For example, some people may have genetic or behavioral predisposition to diabetes, but live normal lives without any symptoms, until some sudden change in diet or lifestyle triggers the disease and requires costly insulin treatment. Even if it then becomes clear that the disease must have been latent for many years, the

¹The courts sometimes extend the notion of "material fact" to opinions such as the prognosis of a specialist; see Clarke (1997, pp. 596–7).

insurer cannot allege bad faith on part of the customer. In this paper we extend the Dixit (2000) model further by distinguishing between unknowing and intentional misrepresentation of type.

We allow two different types of investigation involving different costs: (i) of the insured's actual risk type (low or high), and (ii) of signal receipt, that is, whether the insured was aware of his risk type. The law allows the company to rescind the contract, and may also levy a fine on the insured, if the latter investigation is made and shows that the insured when he chose the low-risk contract knew himself to be high-risk. We characterize the optimal contract (the company's probabilistic investigation strategies and promises of payouts conditional on its investigations and their outcomes) subject to the provisions of the law and the usual profit and incentive constraint. Not all features of the contracts that are optimal in our model are observed in reality, and we discuss possible reasons.

A brief statement of our results, with an intuition for the differences between them and the original R-S model, goes as follows. In R-S, information is revealed by the insurance applicants' self-selection. The low-risk types are better able to withstand partial coverage than are high-risk types (the single crossing property); therefore revelation always comes at the cost of partial insurance for the low-risk types. In contrast, in our model the insurance companies can carry out investigations to elicit information, and then reward good-faith disclosures and punish bad-faith misrepresentations. This allows richer contract forms – zero indemnity when bad faith is established, partial coverage when neither good nor bad faith is established (as in R-S), and a reward in the form of over-coverage when good faith is established. To elicit truth-telling, the *gap* between the payouts in the cases of good and bad faith is important; this gap can be achieved at a lower efficiency cost by combining such rewards and punishments.

Verifying the risk type may be used as a first step in a sequential claim handling procedure. If type verification reveals high risk, then the insurer may decide to investigate the policyholder further by performing a signal investigation, to establish whether the misrepresentation of risk was intentional. Alternatively, the company may directly investigate whether the insured had received the signal. We establish the condition under which it

chooses the direct or the sequential procedure. We also show that the optimal signal investigation (direct or sequential) is always random, i.e. performed with probability less than one.

The law of contract usually places the burden of proving bad faith is on the insurer. In Section 4 we suggest that this is usually the most efficient solution, unless the policyholder can prove good faith at a much lower cost.

2 The setting

There is a large population, facing individual risks of accident. All individuals maximize the expected utility $\mathbf{E}[U(W)]$, where W denotes wealth, the utility function U satisfies $U' > 0$ and $U'' < 0$, and $\mathbf{E}[\cdot]$ is the expectation operator. They face an idiosyncratic risk of accident. If no insurance is taken out, we have $W = W_N$ in the no-accident state and $W = W_A$ in the accident state; $A = W_N - W_A$ is the loss from an accident. Individuals differ according to their probability of accident π . We have $\pi = \pi_\ell$ for a low-risk individual (or ℓ -type) and $\pi = \pi_h$ for a high-risk individual (or h -type), where $0 < \pi_\ell < \pi_h < 1$. The fraction of high-risk individuals is λ , with $0 < \lambda < 1$.

Each individual receives a signal $\sigma \in \{b, g\}$ about his risk type: low risk individuals always gets g but high risk individuals get g with probability q and b with probability $1 - q$, with $0 < q < 1$. Then someone who has received the ‘bad’ signal b is sure to be of high-risk h -type:

$$\text{Prob}(h\text{-type} \mid \sigma = b) = 1.$$

Using Bayes’ law, the probability that someone who has received the ‘good’ signal g is nevertheless of the high-risk type, which we denote by α , is

$$\alpha \equiv \text{Prob}(h\text{-type} \mid \sigma = g) = \alpha = \frac{\lambda q}{\lambda q + (1 - \lambda)} \in (0, \lambda).$$

Thus the lower is α , the more precise is the signal.

When an individual gets signal b , he knows he is of h -type and his accident probability

is π_h . When an individual perceives g , his accident probability is

$$\hat{\pi} = \alpha \pi_h + (1 - \alpha) \pi_\ell = \frac{\lambda q \pi_h + (1 - \lambda) \pi_\ell}{\lambda q + (1 - \lambda)}$$

We also find it useful to denote by β the conditional probability that someone who has received the ‘good’ signal g and has incurred an accident is of the high-risk h -type. Using Bayes’ theorem again, we have

$$\beta \equiv \text{Prob}(h\text{-type} \mid \sigma = g, \text{Accident}) = \frac{\pi_h \alpha}{\hat{\pi}} = \frac{\pi_h \alpha}{\pi_h \alpha + \pi_\ell (1 - \alpha)}.$$

Then it is easy to verify that

$$\pi_\ell < \hat{\pi} < \pi_h, \quad \beta > \alpha.$$

Insurance companies are assumed to be risk neutral, they have no administrative cost, and there is free entry in the insurance market. Each company can offer one type of contract. Of course the market may have different types of contracts offered by different companies. It is sufficient to consider two types of contracts, respectively labelled C_h and C_ℓ , designed for the h -types and ℓ -types respectively. No company offering a C_h contract will object if an ℓ -type chooses it, so such contracts will not have any investigation or rescinding provisions. However, a company offering a contract C_ℓ intended for ℓ -types would make a loss if h -types selected it, either accidentally (in good faith) or intentionally (in bad faith). The good-faith principle allows safeguards against this. The company can make an investigation upon receipt of a claim by an individual who has chosen C_ℓ . The company can find out the individual’s true type at cost c_t , and can also verify the perceived signal at cost c_s , either after a type investigation has revealed that the policyholder is a high-risk individual (the sequential procedure) or without verifying the type first (the direct procedure). If a signal investigation reveals that a C_ℓ -policyholder had received b , then he is proved to have been in bad faith. Then the company is allowed to cancel the contract; we assume that it must return the premium, but allowing it to retain the premium causes only minor algebraic changes. We also assume that the courts levy an exogenously specified fine f on the customer. If the policyholder had received g , he was in good faith even though a type investigation may show

him to be h -type. What the contract specifies in this eventuality is determined by the usual optimization and equilibrium considerations; but no fines are levied.

We use the following notation for the contracts:

$$C_h = (k_h, x_h), \quad C_\ell = (k_\ell, x_n, x_\ell^t, x_h^t, x_{hb}^s, x_{hg}^s, x_b^d, x_g^d; p_n, p_t, p_s, p_d)$$

where k_h, k_ℓ are the insurance premiums paid in advance, the various p 's are the probabilities of investigation if a claim is made, and the various x 's are the net payouts to the policyholder² in the various contingencies of investigations and their outcomes. Figure 1 shows the contingencies, the probabilities, and the payouts. All the x 's must be non-negative; the probabilities must all be non-negative, and satisfy $p_n + p_t + p_d = 1$, $0 \leq p_s \leq 1$.

**** FIGURE 1 HERE ****

We assume that the insurance companies commit to these verification strategies.³ We disregard the sequential procedure in which signal investigation comes before type investigation, since it would be suboptimal (see Proposition 4).

Thus, except for the more general nature of the C_ℓ contracts to allow for verification, our setting directly follows Rothschild and Stiglitz (1976). Our notion of equilibrium is also the same as theirs. As explained by Kreps (1990, p. 649), this is a subgame perfect Nash equilibrium of the following game. At the first stage, each of the large number of potential insurance companies decides whether to offer a contract, and if so, chooses the type and specification of its contract. At the second stage, each consumer chooses whether to accept a contract, and if so, what contract. Then each consumer's outcome (accident or no accident) is realized, those who have suffered accidents file claims, the insurance companies follow through with their committed investigation strategies, and depending on the outcome of the investigations that may be carried out, pay the appropriate indemnities.

Let s_b be the probability for an individual who has perceived $\sigma = b$ to take out the C_ℓ contract – this is the probability of acting in bad faith – and let s_g be the probability for

²So $k_h + x_h, k_\ell + x_n$ etc. are the gross indemnity amounts.

³This is problematic: in equilibrium no one acts in bad faith, so the company is tempted ex post to save the cost of the investigation. Picard (2002) relaxes the assumption of commitment and finds that semi-separating equilibria can arise, where customers randomize.

an individual who has perceived g to choose the C_h contract, with $0 \leq s_b, s_g \leq 1$. The full insurance contracts with fair premium are denoted by C_h^* and C_ℓ^* , respectively for h -types and for ℓ -types, that is $C_h^* = (k_h^*, x_h^*) = (\pi_h A, A - \pi_h A)$ and $C_\ell^* = (k_\ell^*, x_\ell^*) = (\pi_\ell A, A - \pi_\ell A)$.

For the same reasons as in Rothschild-Stiglitz (1976) and Dixit (2000), there cannot be a pooling equilibrium where just one type of contract is on offer. The following definition characterizes a candidate separating equilibrium:

Definition 1 *A candidate equilibrium of the insurance markets consists of two contracts $C_\ell = (k_\ell, x_n, x_\ell^t, x_h^t, x_{hb}^s, x_{hg}^s, x_b^d, x_g^d, p_n, p_t, p_s, p_d)$ and $C_h = (k_h, x_h)$ offered by different insurers, and contract choice strategies s_b, s_g such that:*

(1) s_b maximizes the expected utility of individuals who have received signal b and s_g maximizes the expected utility of individuals who have received g , given (C_ℓ, C_h) .

(2) Insurers make nonnegative profit at equilibrium.

(3) No company can offer a contract $C'_\ell \neq C_\ell$ or $C'_h \neq C_h$ (where C'_ℓ is labelled as available only to low-risk individuals), which, offered in addition to C_ℓ, C_h would make positive profits, assuming that individuals make optimal choice among this enlarged menu of contracts.

3 Equilibrium contract and verification

We now proceed to analyze when this candidate equilibrium is actually an equilibrium, and obtain its properties. We begin with a characterization result:

Proposition 1 *In a market equilibrium, we have*

(i) $s_b = s_g = 0$

(ii) $C_h = C_h^*$

(iii) C_ℓ maximizes

$$\begin{aligned} & (1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}[p_n U(W_A + x_n) + p_t(1 - \beta)U(W_A + x_\ell^t) \\ & + \beta p_t(1 - p_s)U(W_A + x_h^t) + \beta p_t p_s U(W_A + x_{hg}^s) + p_d U(W_A + x_g^d)] \end{aligned} \quad (1)$$

subject to

$$U(W_N - \pi_h A) \geq (1 - \pi_h) U(W_N - k_\ell) + \pi_h [p_n U(W_A + x_n) \quad (2)$$

$$+ p_t(1 - p_s) U(W_A + x_h^t) + p_t p_s U(W_A + x_{hb}^s - f) \\ + p_d U(W_A + x_b^d - f)] \quad (3)$$

$$(1 - \hat{\pi}) k_\ell - \hat{\pi} [p_n x_n + (1 - \beta) p_t x_\ell^t + \beta p_t (1 - p_s) x_h^t \\ + \beta p_t p_s x_{hg}^s + p_d x_g^d + p_t c_t + (\beta p_t p_s + p_d) c_s] \geq 0 \quad (4)$$

$$x_n, x_\ell^t, x_h^t, x_{hb}^s, x_{hg}^s, x_b^d, x_g^d, p_t, p_d, p_n \geq 0, \quad p_t + p_d + p_n = 1, \quad 0 \leq p_s \leq 1.$$

(iv) *The good faith principle enlarges the set of parameter values for which this equilibrium exists. When equilibrium exists both with and without the principle, the one without the principle is the same as the Rothschild-Stiglitz equilibrium, and the one with the principle is Pareto superior.*

The proof is the same as that in Rothschild and Stiglitz (1976) and Dixit (2000). Since there are two information sets, the equilibrium has the same structure as the standard R-S equilibrium: it is separating, with full insurance at fair price for b -recipients. Competition ensures that C_ℓ maximizes the expected utility of g -recipients subject to the b -recipients' self-selection constraint and to the company's no-expected-loss constraint. The reason for the possible non-existence of the equilibrium is that a pooling contract can give everyone higher expected utility, specifically, g -recipients do better than with C_ℓ . If the good faith principle did not apply, the maximization problem solved by C_ℓ would be more constrained because the x 's could not be made contingent on an investigation. Therefore non-existence could arise for more parameter values. Pareto superiority follows from the fact that the less-constrained problem yields a higher value of the objective function (expected utility of the g -recipients). And since, in the absence of the good-faith principle, the findings of an investigation cannot be used, the company would not make a costly investigation. Therefore the equilibrium without the principle, if it exists, must be the same as in R-S.

Proposition 2 characterizes the payout to a policyholder who has received signal g and chosen contract C_ℓ .

Proposition 2 $0 = x_{hb}^s = x_b^d \leq x_h^t < x_n < A - k_\ell < x_\ell^t = x_{hg}^s = x_g^d$.

The formal proofs of this and the subsequent propositions are in the Appendix; the text gives the intuitive arguments. x_{hb}^s or x_b^d is paid to policyholders when bad faith has been proved: for obvious incentives reasons it is optimal to enforce the highest possible penalty in this case, i.e. canceling the contract and returning the premium which correspond to $x_{hb}^s = x_b^d = 0$. Next, x_h^t is a low payment for being shown to be a high-risk type even though there is no investigation of signal receipt (bad faith), and x_n is an intermediate settlement if no investigation is carried out at all. $A - k_\ell < x_\ell^t$ shows that there is overinsurance when a risk-type investigation reveals that the policyholder is a ℓ -type. Overinsurance is a reward for truthtelling, since a ℓ -type always perceives g . However, we also have $x_\ell^t = x_{hg}^s = x_g^d$ because for insurance reasons it is optimal not to punish the h -types who have received g (whether the fact that the individual is a high risk has been established or not). In other words, when good faith is established, high-risk individuals should not be penalized by comparison with low-risk types.

Insurance contracts in reality do not usually have so many different levels of payoffs contingent on the company's investigation. Companies usually do make low initial offers without any investigation. Excessive payouts if investigations reveal good faith are not common. However, experience-rating in dynamic contexts may serve a similar purpose. It is also possible that the designers of insurance contracts have simply not yet realized the usefulness of rewards as an additional instrument for separation purposes.

Proposition 3 *Ceteris paribus, there exists $\bar{\alpha} > 0$ such that $x_h^t = 0$ when $0 < \alpha < \bar{\alpha}$.*

Proposition 3 states that, when the signal is precise enough (i.e. α is small enough), no insurance indemnity should be paid to a policyholder who has chosen C_ℓ and who proves to be a h -type, after a type investigation (but no signal investigation) has been carried out. This makes intuitive sense – if the signal is sufficiently precise, then the revelation that a C_ℓ -chooser has h -type is close to being proof of bad faith. The difference is that the law does not allow zero indemnity for being close to proof; it is merely efficient for the voluntary private contract to make this provision.

Proposition 4 *A sequential procedure with direct investigation of signal followed by investigation of risk type is not optimal.*

The intuition is as follows. If signal investigation reveals bad faith, then the type of the policyholder is known to the company, and type investigation is unnecessary. If signal investigation reveals good faith, then the type of the policyholder is still unknown to the company (as well as to the policyholder), but optimal insurance should lead to the same payout, whether the individual is type- h or type- ℓ . In either case, investigation of type would be useless, and being costly, should not be done.

Proposition 5 *In the optimal verification strategy, $p_s p_t < 1$, and $p_d < 1$.*

Proposition 5 means that the optimal verification strategy is random. It is not optimal always to verify the risk type and the signal in the sequential procedure, or the signal in the direct procedure. Investigating the signal (either directly or sequentially) is used as an incentive device to deter b -recipients from being dishonest. However, sure investigation would go too far: the b -recipients self-selection constraint would be met with some slack. A small decrease in the signal verification probability p_s or p_d would decrease the insurance cost but still keep the self-selection constraint satisfied.

Proposition 6 *If $c_s(1 - \beta) < c_t$ and $p_t > 0$, then $p_s = 0$. If $c_s(1 - \beta) > c_t$, then $p_d = 0$*

The direct and the sequential investigation procedures are alternatives, and only the less costly of the two is chosen. Direct verification of signal costs c_s . Verification of the signal costs c_t ; the signal need be verified sequentially only if the type is seen to be high, and the probability of this contingent on an accident is β . Thus the total expected cost of the sequential procedure is $c_t + \beta c_s$. Proposition 6 says that insurers only use the less costly of the two procedures for signal verification.

4 The onus of proof

We have so far assumed that the law places the burden of proof on the insurance company when it alleges bad faith. Now we examine briefly the difference made if the burden is on the insured to prove good faith. For simplicity, we suppose that the insurance payout cannot depend on the information gathered by the insurer through investigation, except when bad faith of the policyholder is proved in which case the contract may be rescinded. In our model, this means $x_h^t = x_n = x_\ell^t = x_{hg}^s = x_g^d$. Let x_ℓ denote this common payout. Also, $x_{hb}^s = x_b^d = 0$.

First consider the case where the burden of proving bad faith is on the insurance company. From Proposition 1, the insurance policy offered to g -recipients at the market equilibrium is obtained by maximizing

$$(1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}U(W_A + x_\ell)$$

with respect to $k_\ell, x_\ell, p_t, p_s, p_d \geq 0$ subject to the b -recipients' self-selection constraint

$$\begin{aligned} U(W_N - \pi_h A) \geq & (1 - \pi_h)U(W_N - k_\ell) + \pi_h \{ [1 - (p_d + p_t p_s)]U(W_A + x_\ell) \\ & + (p_d + p_t p_s)U(W_A - f) \} \end{aligned}$$

and the no-expected loss constraint

$$(1 - \hat{\pi})k_\ell - \hat{\pi}[x_\ell + p_t c_t + (\beta p_t p_s + p_d)c_s] \geq 0,$$

with $p_t + p_d \leq 1$ and $p_s \leq 1$. Let us call this maximisation ‘‘Problem P1’’.

Now consider the alternative legal system where the onus of the proof rests on the policyholder. Assume type investigation has revealed high risk and that the policyholder has to prove good faith in court to receive the insurance indemnity. Let D denote the costs to a g -recipient to prove good faith in court, which may cover monetary expenditures as well as time spent in legal proceedings. Now the company only investigates the type. If type h is revealed, and the policyholder is unable to prove good faith in court, he has to pay the fine f and the contract is rescinded. We assume that D is small enough and f is large enough that proving good faith in court to be the optimal strategy of g -recipients. This is the case

if $x_\ell > D - f$ which is assumed here. Then the policy offered to g -recipients at the market equilibrium is obtained by maximizing

$$(1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}[(1 - p_t\beta)U(W_A + x_\ell) + p_t\beta U(W_A + x_\ell - D)]$$

with respect to $k_\ell, x_\ell, p_t \geq 0$ subject to

$$U(W_N - \pi_h A) \geq (1 - \pi_h)U(W_N - k_\ell) + \pi_h[(1 - p_t)U(W_A + x_\ell) + p_t U(W_A - f)],$$

and

$$(1 - \hat{\pi})k_\ell - \hat{\pi}(x_\ell + p_t c_t) \geq 0,$$

with $p_t \leq 1$. Call this maximisation ‘‘Problem P2’’.

Welfare maximisation requires that the onus of the proof rests with the insurer when the expected utility of g -recipients is higher in Problem P1 than in Problem P2 and the onus of the proof should rest with the policyholder in the other case. Proposition 7 characterizes the threshold value of D above which the onus of the proof should rest on the company.

Proposition 7 *The onus of the proof should rest on the insurance company when $D > D^*$ and on the policyholder when $D < D^*$, where the threshold D^* satisfies $D^* < \inf\{c_s, (c_s - c_t)/\beta\}$. In particular, when $D = c_s$ the onus of the proof should rest on the insurer.*

The intuition for Proposition 7 is that the cost to policyholders of establishing good faith cannot be insured. Hence, individuals run the risk of having to prove good faith. When the cost to the insurer of proving bad faith is not larger than the cost to the policyholder of proving good faith, i.e. when $\inf\{c_s, (c_s - c_t)/\beta\} \leq D$, then the onus of the proof should rest with the insurer. This is also true if D is slightly larger than $\inf\{c_s, (c_s - c_t)/\beta\}$, hence the result on the threshold D^* . This suggests that the usual legal practice of placing the burden of proving bad faith on the insurer may be the most efficient one, unless the insured’s cost of proving good faith is especially low.

APPENDIX

Proof of Proposition 2

Let \mathcal{L} be the Lagrangean and let μ_1 , μ_2 and θ be Lagrange multipliers for (2), (3) and (4) respectively, with $\mu_1, \mu_2 \geq 0$. The expressions for the derivatives of the Lagrangean with respect to the choice variables are:

$$\partial \mathcal{L} / \partial k_\ell = [\mu_1(1 - \pi_h) - (1 - \hat{\pi})]U'(W_N - k_\ell) + \mu_2(1 - \hat{\pi}) \quad (5)$$

$$\partial \mathcal{L} / \partial x_n = p_n \{[\hat{\pi} - \mu_1 \pi_h]U'(W_A + x_n) - \mu_2 \hat{\pi}\} \quad (6)$$

$$\partial \mathcal{L} / \partial x_\ell^t = p_t(1 - \beta) \{ \hat{\pi} U'(W_A + x_\ell^t) - \mu_2 \hat{\pi} \} \quad (7)$$

$$\partial \mathcal{L} / \partial x_h^t = p_t(1 - p_s) \{ [\beta \hat{\pi} - \mu_1 \pi_h]U'(W_A + x_h^t) - \mu_2 \hat{\pi} \beta \} \quad (8)$$

$$\partial \mathcal{L} / \partial x_{hg}^s = p_t \beta p_s [\hat{\pi} U'(W_A + x_{hg}^s) - \mu_2 \hat{\pi}] \quad (9)$$

$$\partial \mathcal{L} / \partial x_{hb}^s = -\mu_1 \pi_h p_t p_s U'(W_A + x_{hb}^s - f) \quad (10)$$

$$\partial \mathcal{L} / \partial x_b^d = -\mu_1 \pi_h p_d U'(W_A + x_b^d - f) \quad (11)$$

$$\partial \mathcal{L} / \partial x_g^d = p_d [\hat{\pi} U'(W_A + x_g^d) - \mu_2 \hat{\pi}] \quad (12)$$

$$\partial \mathcal{L} / \partial p_t = \hat{\pi} \{ (1 - \beta)U(W_A + x_\ell^t) + \beta(1 - p_s)U(W_A + x_h^t) + \beta p_s U(W_A + x_{hg}^s) \} \quad (13)$$

$$\begin{aligned} & -\mu_1 \pi_h \{ (1 - p_s)U(W_A + x_h^t) + p_s U(W_A + x_{hb}^s - f) \\ & -\mu_2 \hat{\pi} \{ (1 - \beta)x_\ell^t + \beta(1 - p_s)x_h^t + \beta p_s x_{hg}^s + c_t + \beta p_s c_s \} - \theta \end{aligned}$$

$$\partial \mathcal{L} / \partial p_s = \hat{\pi} \beta p_t [U(W_A + x_{hg}^s) - U(W_A + x_h^t) - \mu_2 (x_{hg}^s - x_h^t)] \quad (14)$$

$$-\hat{\pi} \mu_2 \beta p_t c_s + \mu_1 \pi_h p_t [U(W_A + x_h^t) - U(W_A + x_{hb}^s - f)]$$

$$\partial \mathcal{L} / \partial p_d = \hat{\pi} [U(W_A + x_g^d) - \mu_2 x_g^d] - \mu_1 \pi_h U(W_A + x_b^d - f) - \mu_2 \hat{\pi} c_s - \theta \quad (15)$$

$$\partial \mathcal{L} / \partial p_n = [\hat{\pi} - \mu_1 \pi_h]U(W_A + x_n) - \mu_2 \hat{\pi} x_n - \theta \quad (16)$$

Note that if an investigation probability is zero, then the net payments corresponding to the outcome of such an investigation are irrelevant and can be set equal to the value determined formally from the first-order conditions which sets the derivatives of the Lagrangean with respect to that net payment equal to zero.

First we prove that $\mu_1 > 0$. Assume the opposite, namely $\mu_1 = 0$. Then (5)-(12) give $x_h^t = x_n = A - k_\ell = x_\ell^t = x_{hg}^s = x_g^d$. Using (13), (15), and (16) then yield $\partial\mathcal{L}/\partial p_t < \partial\mathcal{L}/\partial p_n$ and $\partial\mathcal{L}/\partial p_d < \partial\mathcal{L}/\partial p_n$, which give $p_t = p_d = 0$ and $p_n = 1$. (3) then implies $x_n = (1 - \hat{\pi})A$ and $k_\ell = \hat{\pi}A$, which contradicts (2) since $\hat{\pi} < \pi_h$. Hence $\mu_1 > 0$.

Next, $\partial\mathcal{L}/\partial x_{hb}^s < 0$ and $\partial\mathcal{L}/\partial x_b^d < 0$ give $x_{hb}^s = x_b^d = 0$.

To prove $x_n > 0$, assume the contrary, that is, $x_n = 0$. Using (6) then gives

$$(\hat{\pi} - \mu_1\pi_h) U'(W_A) - \mu_2\hat{\pi} \leq 0$$

Since $\beta < 1$, this implies

$$\left(\hat{\pi} - \frac{\mu_1\pi_h}{\beta}\right) U'(W_A) - \mu_2\hat{\pi} < 0$$

Hence $\partial\mathcal{L}/\partial x_h^t < 0$ if $p_t > 0$ and $0 < 1 - p_s < 1$; this gives $x_h^t = 0$. In such a situation, (2) would not be binding which gives $\mu_1 = 0$, hence a contradiction. We deduce that $x_n > 0$ and $0 < \mu_1 < \hat{\pi}/\pi_h$.

Setting the derivatives of the Lagrangean with respect to $k_\ell, x_n, x_\ell^t, x_h^t, x_{hg}^s$ and x_g^d equal to zero then gives

$$\begin{aligned} U'(W_A + x_h^t) &= \frac{\mu_2\hat{\pi}\beta}{\hat{\pi}\beta - \mu_1\pi_h} > \frac{\mu_2\hat{\pi}}{\hat{\pi} - \mu_1\pi_h} = U'(W_A + x_n) \\ &> \frac{\mu_2(1 - \hat{\pi})}{(1 - \hat{\pi}) - \mu_1(1 - \pi_h)} = U'(W_N - k_\ell) \\ &> \mu_2 = U'(W_A + x_\ell^t) = U'(W_A + x_{hg}^s) = U'(W_A + x_g^d) \end{aligned}$$

when $x_h^t > 0$, which gives

$$W_A + x_h^t < W_A + x_n < W_N - k_\ell < W_A + x_\ell^t = W_A + x_{hg}^s = W_A + x_g^d$$

Proof of Proposition 3

Let $\mu_1(\alpha)$ be the value of μ_1 written as a function of α , everything else being held unchanged in the model. We know from the proof of Proposition 2 that $\mu_1(\alpha) > 0$ for all α in $(0, 1)$. Let us show that there exist $\mu_1^* > 0$ and $\alpha^* > 0$ such that $\mu_1(\alpha) > \mu_1^*$ when $\alpha < \alpha^*$.

Suppose that is not the case. Then we can find a sequence α^n , $n = 1, 2, \dots$ such that α^n and $\mu_1^n = \mu_1(\alpha^n)$ simultaneously go to zero when n go to infinity. Let C_ℓ^n be the corresponding sequence of equilibrium C_ℓ contracts. Note that $\hat{\pi}$ goes to π_ℓ when α goes to zero. When n goes to infinity, C_ℓ^n converges toward the full insurance contract C_ℓ^* with $p_t = p_s = 0$, which contradicts the self-selection constraint (2). Hence, when $\alpha < \bar{\alpha} = \inf\{\alpha^*, \mu_1^*\}$, we have $\alpha < \mu_1(\alpha)$ which gives $\hat{\pi}\beta = \alpha\pi_h < \mu_1\pi_h$. In such a case, we have $\partial\mathcal{L}/\partial x_h^t < 0$, which gives $x_h^t = 0$.

Proof of Proposition 4

Assume that the risk type is investigated with probability δ when direct verification of signal has revealed that the policyholder has perceived g . Then in the expected utility of g -recipients, $p_d U(W_A + x_g^d)$ is replaced by

$$p_d(1 - \delta)U(W_A + x_g^d) + \delta\alpha p_d U(W_A + x_{gh}^d) + \delta(1 - \alpha)p_d U(W_A + x_{g\ell}^d)$$

where x_{gh}^d and $x_{g\ell}^d$ respectively denote the net payout when investigation reveals high risk and low risk. Nothing is changed in the b -recipients' self selection constraint. In the firm's no-expected loss constraint, $p_d x_g^d$ is replaced by

$$p_d(1 - \delta)x_g^d + \delta\alpha p_d x_{gh}^d + \delta(1 - \alpha)p_d x_{g\ell}^d$$

and $p_d c_s$ is replaced by $p_d(c_s + \delta c_t)$. Using

$$\partial\mathcal{L}/\partial x_g^d = \partial\mathcal{L}/\partial x_{gh}^d = \partial\mathcal{L}/\partial x_{g\ell}^d = 0$$

gives

$$x_g^d = x_{gh}^d = x_{g\ell}^d$$

We deduce

$$\partial\mathcal{L}/\partial\delta = -\mu_2 p_d c_t < 0$$

establishing that we have a corner solution with $\delta = 0$.

Proof of Proposition 5

If $p_s p_t = 1$ or $p_d = 1$ then the expected utility of a b -recipient from claiming to have perceived g would be

$$(1 - \pi_h)U(W_N - k_\ell) + \pi_h U(W_A - f)$$

which is less than from buying no insurance and, a fortiori, less than from buying the C_h^* contract. So the incentive constraint is slack which contradicts $\mu_1 > 0$.

Proof of Proposition 6

We can simplify (13)-(16) by observing that these formulae involve many expressions of the form

$$K U(W_A + x) - H x,$$

where the x is defined by

$$K U'(W_A + x) = H.$$

Hence in the derivatives of the Lagrangean with respect to investigation probabilities, we may use

$$K U(W_A + x) - H x = H \Phi(x)$$

where

$$\Phi(x) = \frac{U(W_A + x)}{U'(W_A + x)} - x.$$

Without loss of generality, we normalize utility so that $U(W_A - f) = 0$. Then $\Phi(0) > 0$ and

$$\Phi'(x) = - \frac{U(W_A + x)U''(W_A + x)}{[U'(W_A + x)]^2} > 0$$

Also define $\theta' = \theta/\mu_2\hat{\pi}$ and $x_g = x_\ell^t = x_{hg}^s = x_g^d$. Then

$$\partial\mathcal{L}/\partial p_t = \mu_2\hat{\pi}\{[1 - \beta(1 - p_s)]\Phi(x_g) + \beta(1 - p_s)\Phi(x_h^t) - c_t - \beta p_s c_s - \theta'\} \quad (17)$$

$$\partial\mathcal{L}/\partial p_s = \mu_2\hat{\pi}\beta p_t[\Phi(x_g) - \Phi(x_h^t) - c_s] \quad (18)$$

$$\partial\mathcal{L}/\partial p_d = \mu_2\hat{\pi}[\Phi(x_g) - c_s - \theta'] \quad (19)$$

$$\partial\mathcal{L}/\partial p_n = \mu_2\hat{\pi}[\Phi(x_n) - \theta'] \quad (20)$$

Now note that

$$\partial\mathcal{L}/\partial p_t \leq 0 \quad (= 0 \text{ if } p_t > 0)$$

$$\partial\mathcal{L}/\partial p_s \leq 0 \text{ if } p_s = 0 \text{ (= 0 if } 0 < p_s < 1, \geq 0 \text{ if } p_s = 1)$$

$$\partial\mathcal{L}/\partial p_d \leq 0 \text{ (= 0 if } p_d > 0)$$

Assume first $c_s(1 - \beta) < c_t$ and $p_t > 0$. Suppose in addition that $p_s > 0$. We have

$$\partial\mathcal{L}/\partial p_t = [1 - \beta(1 - p_s)]\Phi(x_g) + \beta(1 - p_s)\Phi(x_h^t) - c_t - \beta p_s c_s - \theta' = 0$$

and

$$\partial\mathcal{L}/\partial p_s = \Phi(x_g) - \Phi(x_h^t) - c_s \geq 0$$

which implies

$$\Phi(x_g) \geq c_t + \beta c_s + \theta'$$

Using $c_s(1 - \beta) < c_t$ then gives

$$\Phi(x_g) > c_s + \theta'$$

which in turn gives $\partial\mathcal{L}/\partial p_d > 0$. This contradicts the optimality condition on p_d . Hence $p_s = 0$ when $c_s(1 - \beta) < c_t$ and $p_t > 0$.

Similar steps prove that when $c_s(1 - \beta) > c_t$, $p_d = 0$.

Proof of Proposition 7

The optimal expected utility of g -recipients in Problem P2 is decreasing with D and it is less than the expected utility reached in Problem P1 if $D > D^*$. Concavity of U implies

$$(1 - p_t\beta)U(W_A + x_\ell) + p_t\beta U(W_A + x_\ell - D) < U(W_A + x_\ell - p_t\beta D)$$

Hence, the optimal expected utility of g -recipients in Problem P2 is lower than the level that would be reached by maximizing

$$(1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}U(W_A + x_\ell - p_t\beta D)$$

subject to the same constraints as in Problem P2. This modified problem will be called Problem P3.

Suppose first that $c_t < (1 - \beta)c_s$, which implies $p_d = 0$ in Problem P1. Observing that $p_s = 1$ is optimal in this problem, it can then be rewritten as maximizing

$$(1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}U(W_A + x_\ell)$$

with respect to $k_\ell, x_\ell, p_t \geq 0$ subject to

$$U(W_N - \pi_h A) \geq (1 - \pi_h)U(W_N - k_\ell) + \pi_h [(1 - p_t)U(W_A + x_\ell) + p_t U(W_A - f)]$$

$$(1 - \hat{\pi})k_\ell - \hat{\pi}[x_\ell + p_t(c_t + \beta c_s)] \geq 0$$

and $p_t \leq 1$.

Let $x'_\ell = x_\ell + p_t \beta c_s$. With this notation, Problem P1 can then be rewritten as the maximization of

$$(1 - \hat{\pi})U(W_N - k_\ell) + \hat{\pi}U(W_A + x'_\ell - p_t \beta c_s)$$

with respect to k_ℓ, x'_ℓ and p_t subject to

$$U(W_N - \pi_h A) \geq (1 - \pi_h)U(W_N - k_\ell) + \pi_h [(1 - p_t)U(W_A + x'_\ell - p_t \beta c_s) + p_t U(W_A - f)]$$

$$(1 - \hat{\pi})k_\ell - \hat{\pi}(x'_\ell + p_t c_t) \geq 0$$

and $p_t \leq 1$.

Comparing Problems P1 and P3 shows that $D > c_s$ is a sufficient condition for the optimal expected utility of g -recipients in P1 to be higher than that in P3, and therefore *a fortiori* higher than that in Problem P2.

Similar steps show that if $c_t > (1 - \beta)c_s$, a sufficient condition for the optimal expected utility of g -recipients in Problem P1 to be higher than the level reached in Problem P2 is $D > (c_s - c_t)/\beta$.

Hence, we have

$$D^* < c_s \text{ if } c_t < (1 - \beta)c_s$$

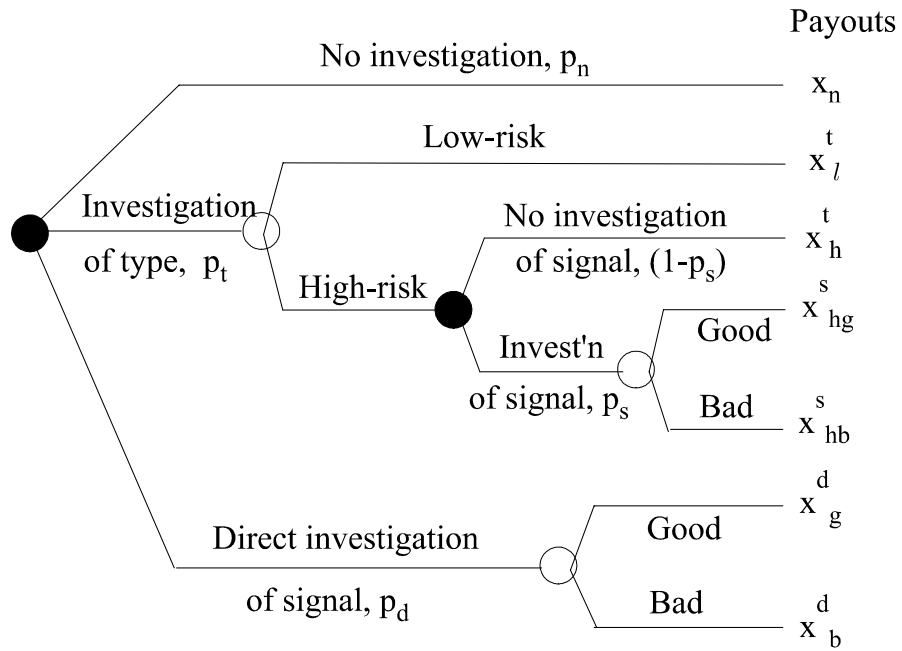
$$D^* < (c_s - c_t)/\beta \text{ if } c_t > (1 - \beta)c_s$$

which may be rewritten as

$$D^* < \inf\{c_s, (c_s - c_t)/\beta\}$$

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- Insurer's action nodes; branches show actions and their probabilities
- Nature's nodes; branches show possible revelations

Figure 1: The Insurer's Randomized Strategies