Costly risk verification without commitment in competitive insurance markets

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Abstract

This paper analyzes the equilibrium of an insurance market where applicants for insurance have a duty of good faith when they reveal private information about their risk type. It is assumed that insurers can, at some cost, verify the type of insureds who file a claim. Insurers are allowed to retroactively void the insurance contract if it is established that the policyholder has misrepresented his (her) risk when the insurance contract was taken out. However, insurers cannot precommit to their risk verification strategy. The paper characterizes the contracts offered at equilibrium, the individuals' contract choice as well as the conditions under which an equilibrium exists.

1 Introduction

Under the law of contracts, an insurer is bound by the provisions of an insurance policy insofar as the policyholder has not deliberately concealed relevant information about his risks when the insurance was taken out. The contract is automatically rescinded in case of risk misrepresentation or non-disclosure of material facts affecting risk unless the policyholder's good faith is established. Indeed, the duty of good faith is a mainstay of the law of insurance contracts : notably, it provides that an applicant for insurance is duty bound to reveal all material information affecting risk. It is this good faith principle that allows the insurer to retroactively void the contract if bad faith is established[1].

Dixit (2000) studies the consequences of the duty of good faith in the setting of a competitive insurance market à la Rothschild-Stiglitz (1976). He shows that the good faith principle achieves a Pareto improvement by allowing the insurers to better separate low risk individuals from high risks ones. If verifying the accident probability is not too costly, then a random ***ex post*** investigation should be carried out when an alleged low risk individual files a claim, no indemnity being paid to a policyholder caught lying. Dixit also shows that a larger insurance indemnity should be paid to a (truthful) low risk individual in case of verification than when the claim is not verified. Furthermore, the good faith principle extends the range of high risk and low risk proportions for which a competitive equilibrium exists. Dixit and Picard (2002) extend Dixit's results to a setting where individuals may have only partial information about their risk level : they only perceive a signal of their risk. Bad faith and good faith then respectively correspond to intentional or unintentional misrepresentation of risk and insurers can verify risk type, perceived signal or both.

In the papers by Dixit (2000) and Dixit-Picard (2002), it is assumed that insurers can commit to a random investigation policy. In their model, all policyholders reveal their information truthfully at equilibrium : in other

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[1] See Clarke (1997) on the duty of good faith in the law of insurance contracts. Colquitt and Hoyt (1997) provide empirical evidence on the importance of good faith clauses in the case of life insurance. Their study uses data from 39 insurers representing 93% of the US life insurance market : in 1994, the total amount of resisted claims disposed of during the year minus the amount paid on those claims was equal to 5.7% of the total death benefits paid during the year. This percentage varies from 0% to 12.4% accross insurers. Most of the reasons provided by insurers for resisting claims were linked to the bad faith of insured, mainly material risk misrepresentation, hidden preexisting condition, misstatement of age or of medical history.

words, they are all in good faith. Consequently, when verification is costly, insurers may be tempted not to verify the policyholders' types or perceived signals with the preannounced frequency, which implies that the insurers' verification strategy is weakened by credibility problems. The present paper will focus attention on this issue of the credibility of the insurers'verification strategy when insurance applicants have a duty of good faith. It will adress the following question : is the good faith principle still valid - as a way to improve market efficiency - when insurers cannot precommit to their risk verification policy?

Credibility may be achieved when insurers and policyholders have infinitely repeated relationships. If the discount rate is not too large, then the benefit associated with a one shot deviation from a preannounced verification strategy will be lower than the loss which results from the inability to commit in the following periods. However, in practice, it is hardly likely that full commitment on a verification strategy can be recovered thanks to repeated relationships. Firstly, the duration of an insurer-policyholder relationship is finite and random. In particular, an increase in the customers' turnover (i.e. a larger probability to quit at each period of time) is equivalent to an increase in the discount rate : a large turnover rate will prevent the insurer to reach a full commitment. Secondly, for a given policyholder the frequency of an accident is usually too low for commitment to be sustainable in a long run relationship. This is all the more likely because the optimal verification strategy is probabilistic which makes the detection of deviations even more difficult. Thirdly, a policyholder usually has imperfect information about the verification frequency of other policyholders which reduces the insurers' ability to build a reputation for frequent auditing.

In this paper, to make the problem more easily tractable, but also to look at the good faith principle from the ***a priori*** less favorable point of view, we will consider a one-shot insurer-policyholder relationship in which any commitment through repeated relationship is dismissed. The setting is similar to the model of Rothschild and Stiglitz (1976). There is a large number of risk averse individuals who have private information on their accident probability : there are low risk individuals and high risk individuals. These individuals seek for insurance on a competitive market. As in Dixit (2000), insurers may carry out a costly verification of the risk type of alleged low risk individuals who file a claim. If investigation reveals that the individual was not truthful, then the good faith principle allows the insurer to canceal the contract and to deny any indemnity. The mixed strategy of a high risk individual is the probability of lying (i.e. of announcing that he or she is a low risk in order to benefit from cheaper insurance). Obviously, a low risk individual always tell the truth. The strategy of each insurer consists of a menu of two contracts offered in the market, one of them being reserved for low risk individuals, and of a risk type verification probability. The remaining player is nature : it chooses the risk type of each individual. Insurers, individuals and nature play a multistage game whose (perfect Bayesian) equilibrium is the market equilibrium. In particular, at equilibrium the insurer's verification probability is the best response to the policyholder's contract choice strategy.

We will show that the good faith principle is still Pareto-improving in this no-commitment setting. We will also establish that an equilibrium exists for a larger set of parameters than in the standard Rothschild-Stiglitz model. The equilibrium may be separating or semi-separating. At a separating equilibrium, different types purchase different contracts : there is full coverage for high risks and partial coverage for low risks and no auditing is implemented at equilibrium. A separating equilibrium in fact coincides with the Rothschild -Stiglitz equilibrium where the high-risk individuals' incentive contract is binding : they are indifferent between buying full insurance at fair premium and choosing the insurance contract which is intended for low-risks individuals. By contrast, at a semi-separating equilibrium, high-risks randomize between both contracts (we may say that they are in bad faith with positive probability) and the risk type is verified with positive probability for alleged low risk individuals who have filed a claim. Furthermore, a separating equilibrium involves partial coverage for low risks individuals (as in the Rothschild-Stiglitz model) but they are overinsured at a semi-separating equilibrium.

There is nothing surprising about the conclusion that the equilibrium is semi-separating when risk types are verified with positive probability. That follows from the fact that the insurers' verification strategy and the insureds' risk misrepresentation strategy are mutual best responses[2]. However, the above-mentioned implications of this characterization are much less straightforward. From a technical point of view, the difficulty of our analysis will arise from the fact that the profitability of contracts - be they offered at equilibrium or in a deviation from equilibrium - must be evaluated at a continuation equilibrium where risk verification probabilities by insurers and

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[2] Picard (1996) analyses a similar situation in the case of claims fraud. In the present paper, after contracts are offered, insurers and individuals play an "inspection game" in which individuals choose a contract and insurers choose an investigation probability. As usual in inspection games (see for instance Fudenberg-Tirole, 1991, p17-18) the equilibrium is in mixed strategy (when some investigation is actually carried out at equilibrium). Note however that in the present model all insurers may not offer the same menu of contracts, particularly in deviation from equilibrium, which complicates matters in a non-trivial way. As shown by Fudenberg and Tirole (1990), the lack of commitment can also lead to semi-separating equilibria in principal-agent problems with moral hazard.

contract choices by individuals are endogenously determined.

Finally, we will also define the conditions of validity of the different regimes that may prevail (separating equilibrium, semi-separating equilibrium or no equilibrium) according to the values of two parameters : the fraction of high risk individuals in the population and the cost of risk type verification.

The paper is organized as follows. Section 2 sets out the model and gives the definition of a market equilibrium with risk verification. Section 3 provides a number of important preliminary properties that hold at a market equilibrium. Section 4 includes our main results, namely the characterization and the conditions of validity of separating and semi-separating equilibria. Section 5 concludes. The main proofs are gathered in appendix.

2 The setting

We consider a large population, facing individual risks of accident. All individuals are risk averse : they maximize the expected utility of wealth $U(W)$; where W denotes wealth and the utility function U is such that $U' > 0$ and $U'' < 0$: They face an idiosyncratic risk of accident. If no insurance policy is taken out, we have $W = W_N$ in the no-accident state and $W = W_A$ in the accident state; $A = W_N - W_A$ is the loss from an accident. Individuals differ according to their probability of accident π and they have private information on their own accident probability. We have $\pi = \pi_\ell$ for a low-risk individual (or ℓ-type) and $\pi = \pi_h$ for a high-risk individual (or h-type) with $0 < \pi_\ell < \pi_h < 1$: The fraction of high-risk individuals is λ with $0 < \lambda < 1$.

Insurance contracts are offered by N insurers (with $N \geq 2$) and we assume that each individual can buy only one contract. An insurance contract is written as $(k; x)$ where k is the insurance premium and x is the net payout in case of an accident. Hence $x + k$ is the indemnity. The expected utility of a policyholder is then written as

$$EU = (1 - \pi)U(W_N - k) + \pi U(W_A + x) \tag{1}$$

where π $\pi_\ell; \pi_h$. Let $C_\ell^* = (k_\ell^*; x_\ell^*) = (\pi_\ell A; A - \pi_\ell A)$ and $C_h^* = (k_h^*; x_h^*) = (\pi_h A; A - \pi_h A)$ be the actuarially fair full insurance contracts, respectively for low risk and high risk.

Let us begin with a brief presentation of the Rothschild-Stiglitz (1976) model. An equilibrium in the sense of Rothschild and Stiglitz consists of a set of contracts such that, when individuals choose contracts to maximize expected utility, **(i)**: Each contract in the equilibrium set makes non-negative expected profit, and **(ii)**: There is no contract outside the equilibrium set that, if offered in addition to those in the equilibrium set, would make strictly positive expected profits. This concept of equilibrium may be understood as a pure strategy subgame perfect equilibrium of a game where insurers simultaneously offer contracts and individuals respond by choosing one of the contracts (or refusing them all). At equilibrium, each contract makes zero profit and there is no profitable deviation at the contract offering stage, given the subsequent reaction of the insurance purchasers.

Rothschild and Stiglitz show that there cannot be a pooling equilibrium where both groups would buy the same contract[3]. Only a separating equilibrium can exist : different types then choose different contracts. Rothschild and Stiglitz establish that the only candidate separating equilibrium is such that high risk individuals purchase full insurance at fair price, i.e. they choose C_h^*; and low risk individuals purchase a contract C_ℓ^{**} with partial coverage. C_ℓ^{**} is the contract that low risk individuals most prefer in the set of (fairly priced) contracts that do not attract high risk individuals. We may write

$$C_\ell^{**} = (k_\ell^{**}; x_\ell^{**}) = (\pi_\ell A^0; A^0 - \pi_\ell A^0) \text{ with } 0 < A^0 < A$$

where A^0 is given by

$$U(W_N - \pi_h A) = (1 - \pi_h)U(W_N - \pi_\ell A^0) + \pi_h U(W_A + (1 - \pi_\ell)A^0) \qquad (2)$$

The Rothschid-Stiglitz equilibrium is illustrated in Figure 1, with state-dependent wealth on each axis[4]. W^1 and W^2 respectively denote final wealth in the no-accident state and in the accident state. When a contract $(k; x)$ is purchased, we have $W^1 = W_N - k$ and $W^2 = W_A + x$. Expected utility is then written as

$$EU = (1 - \pi)U(W^1) + \pi U(W^2)$$

The no-insurance situation corresponds to point E with coordinates $W^1 = W_N$ and $W^2 = W_A$. The high risk and low risk fair-odds line go through E, with slopes (in absolute value) respectively equal to $\pi_h=1- \pi_h$ and $\pi_\ell=1-\pi_\ell$: At C_h^* the h-type indifference curve is tangent to the high risk fair-odds line EH. Similarly, C_ℓ^* is at a tangency point between a ℓ-type indifference curve

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[3] Indeed in this case, it would be possible to attract low risk individuals by offering another contract that would be prefered to the initial contract by low risk but not by high risk individuals. Such a contract would be profitable, hence a contradiction with the definition of an equilibrium.

[4] Because no ambiguity may occur, we use the same notation for insurance contracts $(k \; x)$ and their images in the $(W^1 \; W^2)$ plane.

and the low risk fair-odds line EL. C_ℓ^{**} is at the intersection between EL and the high-risk indifference curve that goes through C_h^*. EA in Figure 1 corresponds to the average fair-odds line, whose slope is $\overline{\pi}=1-\overline{\pi}$; with $\overline{\pi}=\lambda\pi_h+(1-\lambda)\pi_\ell$.

Rothschild and Stiglitz also show that the candidate equilibrium $C_h^*; C_\ell^{**}$ is actually an equilibrium (in the sense of the above definition) if and only if λ is larger than a threshold $\hat{\lambda}$, with $\hat{\lambda}$ $(0;1)$. When $\lambda=\hat{\lambda}$; the low-risk indifference curve that goes through C_ℓ^{**} is just tangent to EA. Hence when $\lambda<\hat{\lambda}$ (as represented in Figure 1), there exist contracts that, if offered in addition to $C_h^*; C_\ell^{**}$, would attract high and low-risk individuals and that would make a positive expected profit. Hence, an equilibrium in the sense of Rothschild and Stiglitz only exists if $\lambda\geq\hat{\lambda}$:

Insert Figure 1

The above given definition of an equilibrium assumes that each insurer can only offer one contract. At equilibrium some insurers offer C_h^* and others offer C_ℓ^{**}: When insurers are allowed to offer a menu of contract, which is certainly a more realistic assumption, then the definition of an equilibrium in the sense of Rothschild and Stiglitz consists of a set of menus that break even on average, such that there is no menu of contracts outside the equilibrium set that, if offered in addition, would make strictly positive expected profits. At an equilibrium, the menu C_h^*,C_ℓ^{**} is offered by all insurers: h-types choose C_h^* and ℓ-types choose C_ℓ^{**}. Hence the set of equilibrium contracts is unchanged. However, the possibility of offering a menu increases the critical proportion of high risk individuals above which an equilibrium exist: there exists λ^* in $(0;1)$, with $\lambda^*>\hat{\lambda}$ such that an equilibrium exists if and only if $\lambda\geq\lambda^*$[5].

In what follows, we will modify the Rothschild-Stiglitz model by considering the consequences of the good faith principle. Applicants for insurance have a duty of good faith, which stipulates that they should reveal their risk type truthfully and provides that if an investigation reveals that a high risk individual passed himself off as a low risk, then the insurance contract may be rescinded. It will be assumed that no third party can verify whether a risk type investigation has actually been carried out, except when risk misrepresentation has been established. In other words, only the proof of risk misrepresentation is verifiable information. Under this assumption, a supposedly low-risk policyholder receives the same insurance indemnity when the truthfulness of his assertion has been verified by the insurer and when

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[5] See Crocker and Snow (1985) and Henriet-Rochet (1990). The fact that λ^* is larger than $\hat{\lambda}$ was pointed out by Rothschild and Stiglitz (1976) themselves.

no investigation has been carried out[6]. In such a framework, an insurance contract is still written as $(k; x)$ where k is the insurance premium and x is the net payout in case of an accident, and the expected utility of a truthful policyholder is given by (1) . No insurance indemnity should be paid to a policyholder who has been caught lying.

At the equilibrium of the insurance market, a contract $C_\ell = (k_\ell; x_\ell)$ is offered to low-risk individuals only, while another contract $C_h = (k_h; x_h)$ may be taken out by any individual whatever his or her type. In case of an accident, a policyholder who has taken out the C_ℓ contract will be investigated with probability $p \quad [0; 1]$: Through investigation, the insurer gets information about the type of the policyholder. This information is verifiable if the policyholder has misrepresented his or her type in which case the insurer voids the contract, which means that no indemnity is paid and the premium is refunded to the policyholder. Verifying the insureds' type costs c to the insurer.

Let s_h (s_ℓ) be the probability for a h-type $(\ell$-type$)$ to take out the C_h (C_ℓ) contract, with $0 \leq s_h; s_\ell \leq 1$: Given C_ℓ and C_h; $(s_h; s_\ell)$ is the strategy of policyholders (conditionally on their type) and p is the strategy of the insurers.

The definition of an equilibrium of the insurance market given hereafter extends the Rothschild-Stiglitz equilibrium concept to the case of *ex post* verification without commitment on the verification strategy. The offer of any insurer (be it made at equilibrium or in a deviation from the equilibrium) consists of a menu of contracts, one of them being available only to ℓ-types. Insurers do not commit on their verification strategy. More precisely, we define an equilibrium of the insurance market as a perfect Bayesian equilibrium of a five stage game. At stage 1, nature chooses the type of each individual : he (or she) is a type-h with probability λ or a type-ℓ with probability $1 - \lambda$. At the second stage, each insurer decides whether to offer a menu of contracts $(C_\ell; C_h)$ and, if so, he chooses the specification of each contract in the menu. At the third stage, each individual decides whether to accept a contract, and if so, which contracts in the proposed menus. At the fourth stage, for each individual, nature decides whether an accident occurs or not with probability $(\pi_h; 1-\pi_h)$ or $(\pi_\ell; 1-\pi_\ell)$ according to the individual's type. The policyholders who have suffered an accident file a claim. At the fifth stage, the insurers choose whether or not to verify the type of the alleged low-risk individuals who have filed a claim and, depending on the results of the investigation, they pay the indemnity or return the premium.

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[6] This restriction on the set of admissible contracts is in line with the common practice of insurers.

An equilibrium of the insurance market will be defined as a perfect Bayesian equilibrium of this game. For the sake of simplicity, we will restrict attention to the case where all insurers offer the same menu at equilibrium, hence the following definition.

Definition 1 ***An equilibrium of the insurance markets consists of a menu of contracts*** $C_h = (k_h; x_h)$, $C_\ell = (k_\ell; x_\ell)$ ***offered by all insurers and strategies*** $s_h; s_\ell; p$ ***such that:***

- ***(i)*** s_h ***maximizes the expected utility of type-***h ***individuals given*** $(C_h, C_\ell; p)$ ***and*** s_ℓ ***maximizes the expected utility of type-***ℓ ***individuals given*** $(C_h; C_\ell)$:
- ***(ii)*** p ***maximizes the insurers' expected profit given*** s_h ***and*** s_ℓ:
- ***(iii) No other menu of contracts*** $C'_h = (k'_h; x'_h)$; $C'_\ell = (k'_\ell; x'_\ell)$ ***can be created - where*** C'_ℓ ***is labelled as available only to low-risk individuals - that if offered in addition to*** $C_h; C_\ell$ ***would increase the profit of the insurer, assuming firstly that individuals make optimal choice among this enlarged menu of contracts given the type verification probabilities for*** C_ℓ ***and*** C'_ℓ ***and secondly that these verification probabilities maximize the insurers' expected profit given the individuals' choice of contract.***

(i) says that individuals choose their insurance contract optimally, given the offer in the market and the type verification probability for C_ℓ: (ii) says that insurers choose their verification strategy optimally given the contract choice of customers. From (iii), for any deviation from the equilibrium contract proposal, there is a continuation equilibrium that makes the deviation unattractive. In short, an equilibrium is characterized by a menu of contracts from which no insurer has interest to deviate given the endogenous play of the continuation game.

It is important to emphasize that the individuals' contract choice strategy and the insurers' risk verification strategy are functions of the set of menus that are offered. $p; s_h; s_\ell$ denote the strategies that are played at equilibrium: C_h,C_ℓ is then the only available menu of contracts. Appraising the profitability of a deviation at the contract supply stage requires us to calculate the individuals'contract choice and the risk verification strategies on the continuation equilibrium path[7]. By way of illustration, note that the C_ℓ-buyers' audit probability may be modified when a new menu of contracts is offered

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[7] This may be more formally expressed as follows. Let $C^{i} = C^{i}_{h}\ C^{i}_{\ell}$ be the offer of insurer $i \quad 1 \quad n$. Assume that $C^{-i} = C^{-i}_{h}\ C^{-i}_{\ell}$ is another offer, with $C^{-i} =$ if $C^{j} = C^{i}$ for all $j = i$. For notational simplicity, we assume that there are at most

in a deviation from equilibrium. Indeed, at equilibrium p and $(s_h; s_\ell)$ are mutual best responses of insurers and individuals, given the contract supply in the market. When another menu $C_h^0; C_\ell^0$ is offered by a deviant firm, then the continuation equilibrium is characterized by the contract choices of individuals in an enlarged set of contracts $C_h; C_\ell; C_h^0; C_\ell^0$ and by the audit probability for C_ℓ and C_ℓ^0. In particular, the investigation probability for C_ℓ may be affected by this deviation. This is a point we will have to keep in mind in what follows.

Remarks :

(1) A straightforward consequence of (iii) in Definition 1 is that each contract C_h and C_ℓ makes non-negative expected profit at equilibrium (for otherwise insurers could increase profit by dropping the contract in deficit). Hence at equilibrium there is no crosssubsidization between contracts[8]. In fact, Proposition 1 hereafter shows that each contract C_h and C_ℓ makes zero profit at a market equilibrium.

(2) In our definition of an equilibrium, we implicitly assume that all individuals (whatever their risk type) reach expected utility levels which are higher than in the initial situation where no insurance is available. This will be actually the case for equilibrium contracts.

(3) If, at an equilibrium where C_h and C_ℓ are offered, we have $s_\ell = 0$; $s_h = 0$ and $p = 0$ (which may occur if the audit cost is too high to make type verification profitable to the insurers) then there exists another equilibrium where $C_\ell^0 = C_h$ is offered to low-risk individuals only, $C_h^0 = C_\ell$ is offered to all individuals, ℓ-type choose C_ℓ^0, h-type choose C_h^0 and there is no auditing. To avoid this trivial multiplicity of equilibria, we impose the additional condition : $p > 0$ if $s_\ell = s_h = 0$.

3 Preliminary results

Before characterizing the contracts proposal by insurers and the contract choice by individuals, we will state some simple results on the type-investigation strategy which may be carried out at equilibrium. Insurers choose p in order to minimize the expected cost of a claim. They refund the premium but do

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two different offers in the market. Then we may write the investigation strategy for C^i as $\phi(C^i\ C^{-i})$. At equilibrium, all insurers offer the same menu $C = \ C_h\ C_\ell$ and we have $p = \phi(C\ \ )$. The effect of a deviation $C^0 = \ C_h^0\ C_\ell^0$ on investigation for C_ℓ is given by $\phi(C\ C^0)$.

[8] Note however that ***out of equilibrium*** crosssubsidization between contracts is possible. Out of equilibrium crosssubsidization refers to contracts $(C_h^0\ C_\ell^0)$ offered in deviation from equilibrium. In such a deviation, the profit (say) from C_ℓ^0 may susidize the loss from C_h^0.

not pay anything else if investigation reveals that an alleged ℓ-type individual was in fact a h-type. Hence, the expected benefit of investigation is equal to qx_ℓ where q denotes the probability that investigation reveals high risk. Bayes law shows that q is actually given by

$$q = \frac{\lambda\pi_h(1-s_h)\pi_h}{\lambda\pi_h(1-s_h) + (1-\lambda)s_\ell\pi_\ell} \tag{3}$$

when $s_h < 1$ and/or $s_\ell > 0$. At equilibrium, p maximizes $qx_\ell - c$ in $[0,1]$, which leads us to the following characterization of the investigation strategy :

Lemma 1 ***At a market equilibrium where*** C_ℓ ***is chosen by some individuals (i.e.*** $s_h < 1$ ***and/or*** $s_\ell > 0$***), we have*** $p < 1$ ***and***

$$p(qx_\ell - c) = 0. \tag{4}$$

In particular, when $s_h = 1$ ***and*** $s_\ell > 0$***, we have*** $p = 0$.

Lemma 1 is easily established. The insurers'optimal strategy is such that

$$\begin{aligned} p &= 0 \text{ if } qx_\ell < c \\ 0 &\leq p \leq 1 \text{ if } qx_\ell = c \\ p &= 1 \text{ if } qx_\ell > c \end{aligned} \tag{5}$$

which gives (4). Lemma 1 says that the C_ℓ contract involves either probabilistic investigation or no investigation at all, i.e. $p < 1$. Indeed, if $p = 1$, then high-risk individuals would never choose C_ℓ since their contract would be systematically voided should a loss occur. Hence $p = 1$ would give $s_h = 1$ at equilibrium. Using (3)-(5) then yields $q = 0$ and $p = 0$, which is a contradiction. Intuitively, the contract choice by h-types induced by systematic investigation would deprive the insurer from any incentive to carry out such an investigation[9]. Likewise, when C_ℓ is chosen only by low risk individuals - i.e. when $s_h = 1$ and $s_\ell > 0$ -, we have $q = 0$ and not investigating the risk type is optimal, which gives $p = 0$. An immediate consequence of this result is that there is no type verification at a separating equilibrium where all h-type individuals choose C_h and all ℓ-type individuals choose C_ℓ, i.e. where

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[9] Note that the equilibrium investigation strategy is also stochastic when insurers can commit to verifying types with a given probability - see Dixit (2000) and Dixit-Picard (2002). Random auditing is in fact optimal in a broad class of costly state verification models as shown by Townsend (1979) and Mookherjee and Png (1989).

$s_h = s_\ell = 1$: We shall later on find conditions under which such a situation actually emerges as a market equilibrium.

Lemma 1 also says that $qx_\ell = c$ when $0 < p < 1$. Hence, there is a relationship betwen the contract choice by policyholders and the net payout of the C_ℓ contract when some degree of type investigation is actually carried out at equilibrium. This relationship is a direct consequence of (5) and it means that the expected profit of investigation should be equal to the investigation cost for stochastic verification to be an optimal insurers'strategy.

Propositions 1 to 3 provide some additional preliminary results on the equilibrium of the insurance market.

Proposition 1 ***In a market equilibrium, each contract*** C_ℓ ***and*** C_h ***earns zero expected profit.***

Proposition 1 is proved in Appendix 1. It shows that insurers cannot make positive expected profits at equilibrium, for otherwise there would exist another pair of contracts which would increase profit if offered in addition to the existing contracts. Intuitively, these additional contracts would be sufficiently close to the initial contracts for beeing profitable and they would be chosen so that they attract all the individuals, hence the increase in profits.

Hence, Proposition 1 confirms that there is no crosssubsidization between the contracts offered at equilibrium : each of them just breaks even. Of course, this result does not precludes crosssubsidization between risks. A contract crosssubsidizes risks when it is chosen simultaneously by high risk individuals and low risk individuals. Indeed, as we shall see, it is very likely that crosssubsidization between risks occurs at equilibrium.

Proposition 1 allows us to reformulate the definition of an equilibrium of the insurance market. It consists of a menu of contracts C_h, C_ℓ and strategies $s_h; s_\ell; p$ such that:

(i) and **(ii)** of Definition 1 are satisfied,

(iii') each contract makes zero profit

(iv') there does not exist any other menu C'_h, C'_ℓ that would make positive profits (if offered in addition to C_h, C_ℓ**)** when individuals and insurers play equilibrium strategies in the corresponding continuation subgame.

Proposition 2 is more specific about the equilibrium. It shows that ℓ-types always choose the contract which is meant for them, i.e. C_ℓ , while h-types may separate by choosing C_h or randomize between C_h and C_ℓ. In particular, there cannot be a pooling equilibrium where both groups of customers would always buy the same contract.

Proposition 2 *In a market equilibrium,* $s_\ell = 1$ *and* $0 < s_h \leq 1$ *.*

The proof of Proposition 2 runs by eliminating all the other candidates for a market equilibrium than those where low risk individuals choose C_ℓ and high risk individuals either separate by choosing C_h as in the standard Rothschild–Stiglitz model or randomize between C_ℓ and C_h. Three possible cases are contemplated and, in each case, we show that there exists a profitable deviation.

In a first case, some ℓ-types and some h-types choose C_h (i.e. $s_\ell < 1$ and $s_h > 0$) : this kind of equilibrium can be precluded by the same reasoning which shows that a pooling equilibrium is not possible in the standard Rothschild-Stiglitz model. Indeed, there exists C'_h in the neighbourhood of C_h that, if added to C_h and C_ℓ, attracts all the ℓ-types, while the h-types prefer C_h or C_ℓ. The insurer who offers C'_h makes a profit because C_h makes non-negative profit when chosen simultaneously by ℓ-types and h-types and C'_h is in the neighbourhood of C_h. Hence, we can't have an equilibrium.

In a second case, all h-types would prefer C_ℓ to C_h while ℓ-types randomize between C_ℓ and C_h (i.e. $s_\ell > 0$ and $s_h = 0$). Let Π_ℓ denote the expected profit of C_ℓ. It is the difference between expected insurance premiums and the sum of expected indemnity payments and expected verification costs, which are respectively the first, second and third term in the following expression:

$$\begin{aligned} \Pi_\ell &= [\lambda(1-s_h)(1-\pi_h p) + (1-\lambda)s_\ell]k_\ell \\ &-[\lambda\pi_h(1-s_h)(1-p) + (1-\lambda)s_\ell\pi_\ell](x_\ell + k_\ell) \\ &-[\lambda(1-s_h)\pi_h + (1-\lambda)s_\ell\pi_\ell]pc \end{aligned} \tag{6}$$

Using $p(qx_\ell - c) = 0$ drawn from Lemma 1 allows us to simplify (6) as follows

$$\begin{aligned} \Pi_\ell &= [\lambda(1-s_h)(1-\pi_h) + (1-\lambda)s_\ell(1-\pi_\ell)]k_\ell \\ &-[\lambda(1-s_h)\pi_h + (1-\lambda)s_\ell\pi_\ell]x_\ell \end{aligned} \tag{7}$$

Hence, at equilibrium, Π_ℓ is equal to the profit level that would be reached if C_ℓ were chosen by h-types and ℓ-types with the same probability as at equilibrium but without any type verification. Using $s_h = 0$ and $\Pi_\ell = 0$ gives

$$[\lambda(1-\pi_h) + (1-\lambda)s_\ell(1-\pi_\ell)]k_\ell = [\lambda\pi_h + (1-\lambda)s_\ell\pi_\ell]x_\ell$$

When $s_\ell < 1 (s_\ell = 1)$, C_ℓ is located below (on) the average odds line EA in the (W^1, W^2) plane. Hence, when $s_\ell < 1,$ there obviously exist profitable contracts C'_h that would be profitable (without any type verification) if offered in addition to C_h, C_ℓ. This conclusion remains true when $s_\ell = 1$ - i.e. when the equilibrium is a pooling equilibrium where all individuals choose C_ℓ - provided that the ℓ-type indifference curve through C_ℓ is ***not*** tangent to EA. The only critical case is when the ℓ-type indifference curve going through C_ℓ is tangent to EA[10]. In that case, making profits is still possible, but a profitable deviation requires an insurer to offer two contracts, one - C'_h - without type verification is chosen by h-types and the other one - C'_ℓ - is selected by ℓ-types. Such profitable pairs of contracts exist (with an out-of-equilibrium crosssubsidization). In particular, as shown in Figure 2, there exist profitable deviations where C'_ℓ is in the neighbourhood of C_ℓ and C'_h is a full insurance contract. C'_ℓ can be chosen different but arbitrarily close to C_ℓ so that C'_ℓ is the only best choice of ℓ-type individuals. C'_h is the optimal choice for h-types even if C'_ℓ does not entail any type verification. Given these strategies of the policyholders, not verifying the types of C'_ℓ-buyers is an optimal strategy of the insurer who offers this contract. In other words, (C'_h, C'_ℓ) is an incentive compatible deviation. Since C_ℓ breaks even when chosen by all customers, (C'_h, C'_ℓ) makes positive expected profit on aggregate when all ℓ-types select C'_ℓ and all h-types select C'_h[11].

Insert Figure 2

Lastly, in the third case, only h-types choose C_ℓ (i.e. $s_\ell = s_h = 0$) which implies $p > 0$ (see Remark 3 after Definition 1). Then h-types do not receive any insurance with probability p and C_ℓ is enforced with probability $1 - p$. Furthermore, C_ℓ is on the h-types fair-odds line because only h-types choose C_ℓ and C_ℓ makes zero profit at equilibrium. Hence it is possible and profitable to attract h-types by offering them a better contract, which contradicts the definition of a market equilibrium.

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[10] At this stage, we may be tempted to reproduce the standard argument of the Rothschild-Stiglitz model. This argument runs by arguing that a pooling equilibrium (here at C_ℓ) cannot exist because there exist profitable deviations through contracts close to C_ℓ that only attract ℓ-types. This argument is not valid here because the offering of a new contract C'_ℓ affects the beliefs of the non-deviant firms on the risk type of its customers. In particular, if ℓ-types prefer C'_ℓ to C_ℓ and h-types keep on choosing C_ℓ, then $p = 1$ would become an optimal strategy of the non-deviant firms if $x_\ell > c$. In such a case, there is no continuation equilibrium (following the new offer C'_ℓ) where ℓ-types select C'_ℓ and h-types keep on choosing C_ℓ as in the Rothschild-Stiglitz argument. In other words, beliefs about risk type act as a threatening device to prevent deviations that would attract ℓ-types only. Making profit through a deviation requires attracting all the insurance purchasers which is possible only through a menu C'_h, C'_ℓ.

[11] Indeed, in the (W^1, W^2) plane, C'_h is located on a lower h-type isoprofit line than C_ℓ In other words, C'_h shows a lower deficit than C_ℓ when chosen by a h-type individual. This is the reason why the incentive compatible deviation (C'_h, C'_ℓ) is more profitable than the pooling allocation where all individuals select C_ℓ.

Next we establish a result common to the Rothschild-Stiglitz model :

Proposition 3 ***In a market equilibrium,*** $C_h = C_h^*$ ***.***

Indeed, Propositions 1 and 2 simultaneously show that C_h is on the h-types fair odds line. If $C_h = C_h^*$; there exists a profitable deviation C_h^0 which would attract h-types (and possibly ℓ-types).

Propositions 2 and 3 show that we may have either a separating equilibrium with $s_h = s_\ell = 1$ or a semi-separating equilibrium with $0 < s_h < 1$ and $s_\ell = 1$; and in both case $C_h = C_h^*$: In a separating equilibrium, h-types choose C_h^* and ℓ-types choose C_ℓ; while in a semi-separating equilibrium ℓ-types choose C_ℓ and h-types randomize between C_ℓ and C_h^*:

4 Characterization of the market equilibrium

The possibility of a semi-separating equilibrium is the main difference between this model and the standard Rothschild-Stiglitz model, where no risk verification is possible. Proposition 4 characterizes the C_ℓ contract offered at such an equilibrium.

Proposition 4 ***There is a single candidate for a semi-separating equilibrium (i.e. at an equilibrium such that*** $s_\ell = 1$ ***and*** $0 < s_h < 1$)$:$ ***In this candidate equilibrium, we have*** $0 < p < 1$ ***and*** $C_\ell = \widehat{C}_\ell \equiv (\widehat{k}_\ell; \widehat{x}_\ell)$ ***maximizes the expected utility of*** ℓ***-types with respect to*** $x_\ell; k_\ell;$ ***subject to***

$$k_\ell = \phi(x_\ell; c) \equiv \frac{\pi_\ell \pi_h x_\ell^2}{(1 - \pi_\ell)\pi_h x_\ell - c(\pi_h - \pi_\ell)} \tag{8}$$

and $\widehat{C}_\ell$ ***provides overinsurance, i.e.*** $\widehat{x}_\ell + \widehat{k}_\ell > A$: ***At equilibrium, the strategy of*** h***-types is***

$$s_h = 1 - \frac{c(1 - \lambda)\pi_\ell}{\lambda \pi_h(\widehat{x}_\ell - c)} \tag{9}$$

Furthermore $\widehat{C}_\ell$ ***goes to*** C^*_ℓ ***and*** s_h ***goes to one when*** c ***goes to zero.***

The proof of Proposition 4 can be sketched as follows. At a semi-separating equilibrium, p $(0; 1)$ and s_h $(0; 1)$ are mutual best responses. Lemma 1 and $s_\ell = 1$ give $qx_\ell = c$ with

$$q = \frac{\lambda \pi_h (1 - s_h)}{\lambda \pi_h (1 - s_h) + (1 - \lambda)\pi_\ell}$$

which implies

$$s_h = 1 - \frac{c(1-\lambda)\pi_\ell}{\lambda \pi_h (x_\ell - c)} \tag{10}$$

(10) gives the strategy of $h-$types that makes insurers indifferent between investigating and not investigating[12]. Using (7), (10) and $s_\ell = 0$ allows us to write Π_ℓ as a function of (k_ℓ, x_ℓ, c). Equation $k_\ell = \phi(x_\ell, c)$ then defines the set of C_ℓ contracts that make zero expected profit. At a semi-separating equilibrium, the expected utility of ℓ-types is maximized subject to $k_\ell = \phi(x_\ell, c)$, for otherwise a profitable contract could be offered[13]. This maximization problem gives $x_\ell = \hat{x}_\ell$ and $k_\ell = \phi(\hat{x}_\ell, c) \equiv \hat{k}_\ell$. Hence at a semi-separating equilibrium, ℓ-types reach an expected utility $\overline{U}(c)$ with $\overline{U}'(c) < 0$ and $\overline{U}(0) = U(W_N - \pi_\ell A)$. Existence of a semiseparating equilibrium also requires $s_h \in (0,1)$ to be an optimal strategy of high risk individuals. This is the case when p is equal to the investigation probability that makes h-types indifferent between C_h^* and C_ℓ.

In order to understand intuitively why $\hat{C}_\ell$ provides overinsurance, let us observe that an increase in x_ℓ leads to an increase in s_h at the equilibrium of the continuation subgame. Indeed, if x_ℓ increases, then q should decrease (and consequently s_h increases) for qx_ℓ to remain equal to c so that stochastic verification - i.e. $p \in (0,1)$ - remains an optimal strategy of the insurers. This increasing relationship between x_ℓ and s_h is given by equation (10). Hence increasing x_ℓ results in less crosssubsidization between risk types, which is beneficial to low risk individuals. More precisely, by using (7) with $\Pi_\ell = 0$ and $s_\ell = 1$, a simple calculation allows us to write the relationship between the premium k_ℓ and the indemnity $k_\ell + x_\ell$ as

$$k_\ell = \pi_\ell (1+\sigma)(k_\ell + x_\ell)$$

where σ denotes the loading factor which is written as

$$\sigma = \frac{\lambda(1-s_h)(\pi_h - \pi_\ell)}{\pi_\ell[\lambda(1-s_h) + 1 - \lambda]}$$

Hence the higher s_h, the lower the loading factor. Using (10) allows us to write the loading factor as a decreasing function of x_ℓ

$$\sigma = \sigma(x_\ell) \equiv \frac{\lambda c(\pi_h - \pi_\ell)}{\pi_h x_\ell - c(\pi_h - \pi_\ell)} \tag{11}$$

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[12] The proof of Proposition 6 hereafter shows that s_h given by (10) is positive under the conditions of existence of a semi-separating equilibrium.

[13] In such a deviation (as in the candidate equilibrium), all ℓ-types would choose the new contract while h-types would randomize between C_h^* and this new contract.

(11) shows that an increase in x_ℓ entails a decrease in the loading factor. In other words, the marginal price of insurance is decreasing with respect to coverage. When c goes to zero, the loading factor goes to zero and the condition $k_\ell = \phi(x_\ell; c)$ goes to the low risk fair-odds line $k_\ell = \pi_\ell x_\ell/(1-\pi_\ell)$. Consequently, $\hat{C}_\ell$ goes to C_ℓ^*. The cost incurred by ℓ-types because of crosssubsidization is

$$\begin{aligned} \pi_\ell\sigma(x_\ell)(k_\ell + x_\ell) &= \pi_\ell\sigma(x_\ell)[\phi(x_\ell; c) + x_\ell] \\ &= \frac{\lambda c\pi_\ell(\pi_h - \pi_\ell)x_\ell}{(1-\pi_\ell)\pi_h x_\ell - c(\pi_h - \pi_\ell)} \end{aligned}$$

This cost is decreasing with x_ℓ. A graphical argument is helpful to explain why this decreasing relationship between between coverage and the cost of crosssubsidization leads to overinsurance. In Figure 3, the locus PP' is the zero profit line for C_ℓ when h-types randomize C_h^* and C_ℓ. In other words, PP' is the image of the condition $k_\ell = \phi(x_\ell; c)$ in the $W^1; W^2$ plane. On PP', we have $W^1 = W_N - \phi(W^2 - W_A; c)$ and the slope of PP' is such that

$$-\frac{dW^2}{dW^1} = \frac{1}{\partial\phi(W^2 - W_A; c)/\partial W^2} > \frac{\pi_\ell}{1-\pi_\ell}$$

Insert Figure 3

Hence at point F, the slope of PP' (in absolute value) is larger than the slope of the ℓ-type indifference curve. $\hat{C}_\ell$ is located at the tangency of PP' and a ℓ-type indifference curve. To visualize the effect of crosssubsidization and the determination of the optimal contract for ℓ-types, let us assume that C_ℓ is at point F. Then consider an infinitesimal increase in coverage $dx_\ell > 0$, with a compensatory variation in premium $dk_\ell = (\pi_\ell/1 - \pi_\ell)dx_\ell$. At point F, there is full insurance and consequently $(dx_\ell; dk_\ell)$ only entails a second-order effect on ℓ-types' expected utility. However, the increase in x_ℓ leads to a lower crosssubsidization, which gives a first-order increase in the expected profit for C_ℓ. Hence the relative positions of PP' and of the ℓ-type indifference curve : they cross at point F as drawn in Figure 3. Tangency of PP' and a ℓ-type indifference curve is reached at $\hat{C}_\ell$ above the 45° degree line, hence the overinsurance result[14].

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[14] In practice, the insureds'moral hazard may make insurers reluctant to offer such overinsurance contracts. The optimal contract would then trade off the incentives to costly risk verification and the mitigation of insureds' moral hazard. For instance, if we simply impose that claims shouldn't be overpaid, the semi-separating equilibrium is at point F with full coverage of losses.

Let $c^* > 0$ defined by

$$\overline{U}(c^*) = (1 - \pi_\ell)U(W_N - \pi_\ell A^0) + \pi_\ell U(W_A + (1 - \pi_\ell)A^0)$$

When $c = c^*$, then ℓ-type individuals are indifferent between $\widehat{C}_\ell$ and C_ℓ^{**}. As shown in what follows, λ^* and c^* are the critical parameters for the existence of a separating or semi-separating equilibrium. Proposition 5 provides necessary and sufficient conditions for the existence of a separating equilibrium and it shows that such an equilibrium coincides with the Rothschild-Stiglitz equilibrium without any type verification.

Proposition 5 ***There exists a separating equilibrium*** $(C_h; C_\ell; s_h; s_\ell; p)$ ***with*** $s_h = s_\ell = 1$; ***if and only if*** $c \geq c^*$ ***and*** $\lambda \geq \lambda^*$***. The separating equilibrium coincides with the Rothschild-Stiglitz equilibrium and there is no type verification i.e.*** $C_h = C_h^*;\ C_\ell = C_\ell^{**}$ ***and*** $p = 0$:

Intuitively, the proof of Proposition 5 runs as follows. The C_ℓ contract offered at a separating equilibrium necessarily coincides with the standard Rothschild-Stiglitz equilibrium (i.e. $C_\ell = C_\ell^{**}$) for otherwise either C_ℓ would make negative profit or there would exist a profitable contract that would attract ℓ-types without attracting h-types, without any type verification. For $(C_h^*; C_\ell^{**})$ to be an equilibrium, two additional conditions should be met. Firstly, it should be impossible for an insurer to make profit by offering a menu of incentive compatible contracts (i.e. without any type verification) that crosssubsidizes h-types and ℓ-types. Such a menu exists if and only if $\lambda < \lambda^*$. Secondly, the existence of a separating equilibrium requires that insurers cannot make positive profits by offering a menu C_ℓ^0,C_h^0 such that there is some type verification for C_ℓ^0 and h-types randomize between C_h^0 and C_ℓ^0. Hence, in this second kind of deviation, the strategies of h-types and of the insurer who offer C_ℓ^0 are mutually best responses. Such a deviation can simultaneously make positive expected profit and divert ℓ-types from C_ℓ^{**} if $c < c^*$. All things considered, existence of a separating equilibrium imposes as a necessary and sufficient condition that $c \geq c^*$ and $\lambda \geq \lambda^*$.

Proposition 6 ***There exists a semi-separating equilibrium*** $(C_h; C_\ell; s_h; s_\ell; p) = (C_h^*; \widehat{C}_\ell; s_h; 1; p)$ ***with*** $0 < s_h < 1$ ***if and only if*** $c < c^*$ ***,*** $\lambda \geq \lambda^*$ ***or*** $c \leq \widetilde{c}(\lambda)$***,*** $\lambda < \lambda^*$ ***where*** $\widetilde{c}(\lambda)$ ***is increasing over*** $[0; \lambda^*]$ ***with*** $\widetilde{c}(\lambda^*) = c^*$ ***and*** $\widetilde{c}(0) = 0$:

Let us sketch the proof of Proposition 6. Proposition 4 has characterized the single candidate for a semi-separating equilibrium. For this candidate to be truly an equilibrium, it shouldn't be possible to make positive profit by offering an out-of-equilibrium menu of contracts. The characterization given in Proposition 4 has already eliminated all possible deviation C'_h, C'_ℓ where the risk-type of C'_ℓ-buyers is audited with positive probability. Hence, it remains to contemplate incentive compatible deviations, i.e. deviations where the risk type of C'_ℓ-buyers is not investigated. Such a profitable deviation exists if one can find a pair of incentive compatible contracts that make non-negative profit on aggregate and that Pareto-dominate the candidate equilibrium. Two cases should be considered. Assume first $\lambda \geq \lambda^*$. In that first case, maximizing the high risks' expected utility in the set of incentive compatible contracts that break even on aggregate and that yield at least as much expected utility as C_h^* to high risk individuals gives $C_h = C_h^*$ and $C_\ell = C_\ell^{**}$. In other words, the Rothschild-Stiglitz pair of contracts is second best Pareto-efficient when $\lambda \geq \lambda^*$. We know that the low-risk expected utility is $\overline{U}(c)$ at the candidate equilibrium. We also know that low risk individuals prefer C_ℓ^{**} to $\widehat{C}_\ell$ if and only if $c > c^*$. Hence, when $\lambda \geq \lambda^*$, there exists a profitable deviation from the candidate equilibrium if and only if $c > c^*$.

Assume now $\lambda < \lambda^*$. Then maximizing the high risks' expected utility requires to crosssubsidize risks. Furthermore, the lower λ the higher the expected utility that can be reached by low risks in the set of incentive compatible contracts that break even on aggregate. Let $\widehat{U}(\lambda)$ denote this maximal expected utility level, with $\widehat{U}'(\lambda) < 0$. A profitable incentive compatible deviation exists if and only if $\widehat{U}(\lambda) > \overline{U}(c)$, or equivalently if $c > \tilde{c}(\lambda)$ where function $\tilde{c}(\cdot)$ is defined by $\widehat{U}(\lambda) = \overline{U}(\tilde{c})$ with $\tilde{c}'(\lambda) > 0$. Since $\widehat{U}(\lambda^*) = \overline{U}(c^*)$, we have $\tilde{c}(\lambda^*) = c^*$. We also have $\widehat{U}(0) = \overline{U}(0) = U(W_N - \pi_\ell A)$ which gives $\tilde{c}(0) = 0$. Hence the candidate semiseparating equilibrium is truly an equilibrium when $c \leq c^*$ if $\lambda \geq \lambda^*$ or when $c \leq \tilde{c}(\lambda)$ if $\lambda < \lambda^*$.

As shown in Figure 4, there are three possible regimes in the (λ, c) plane : a separating equilibrium regime, a semi-separating equilibrium regime and a regime where there is no equilibrium at all.

Insert Figure 4

If the insurers were not allowed to void the contract when misrepresentation is established, then an equilibrium would exist only if $\lambda \geq \lambda^*$: this setting corresponds to the standard Rothschild-Stiglitz model. Allowing the insurers to void the contract enlarges the set of parameters for which an equilibrium exists. More precisely, the smaller the verification cost c, the smaller the threshold for λ above which an equilibrium exists. Equivalently, for any λ, an equilibrium always exists if c is small enough.

If c were equal to zero, there would not be effectively any uncertainty on the insureds' risk type and competition on the insurance market would lead to type separation and full insurance at fair price. As shown in Proposition 4, when c goes to zero, then $\hat{C}_\ell$ goes to C^*_ℓ and s_h goes to $\mathbf{1}$ in the semi-seprating equilibrium, without any discontinuity at $c = 0$. On the contrary, there is a discontinuity in insurance coverage and premium when c reaches the threshold c^* since we go from partial coverage in the separating equilibrium regime to overinsurance in the semi-separating equilibrium regime.

Lastly, we may note that the semi-separating equilibrium (when it exists) Pareto-dominates the Rothschild-Stiglitz equilibrium since the welfare of high risk individuals is increased while the low risks' expected utility is unchanged. Hence, although insurers cannot commit on their verification strategy, allowing them to void the contract improves efficiency in the market and makes existence of equilibrium more likely.

5 Conclusion

The good faith principle is a major pillar of the law of insurance contracts. It states that insureds have a duty of good faith and it allows insurers to rescind contracts ***ex post*** when intentional misrepresentation of risk is established. Thereby it contributes to more efficient risk sharing in insurance markets under asymmetric information. However the effects of the good faith principle mat conceivably be weakened or even cancelled by a credibility constraint on the insurers' risk verification strategy.

In order to better understand the effects of this credibility constraint, we have analyzed the equilibrium of an insurance market where applicants for insurance have a duty of good faith when revealing their risk type and insurers cannot precommit to their risk verification policy. Three main results have been reached. Firstly, the equilibrium qualitatively differs from the one that prevails in the standard Rothschild-Stiglitz model : here it may be either separating or semi-separating. At a semi-separating equilibrium, there is some degree of bad faith from high risk individuals : they do not always reveal their risk type truthfully. Furthermore, low risk individuals get overinsurance at a semiseparating equilibrium, contrary to the main prediction of the standard Rothschild-Stiglitz model. Secondly, the possibility of cancealing the contract when bad faith is established extends the set of parameters for which a competitive equilibrium exists. In particular, an equilibrium always exists if the verification cost is low enough. Thirdly, the good faith principle remains Pareto-improving in comparison with the Rothschild - Stiglitz equilibrium, although insurers are deprived of any possibility of precommitment in their risk verification strategy.

Note finally that, among various possible extensions of our analysis, modifying the game structure of our model would allow us to contemplate other equilibrium benchmarks (i.e. equilibrium without any possibility of type verification) than the Rothschild-Stiglitz equilibrium. This would allow us to appraise the robustness of our conclusion on the efficiency gains that are attributable to the good faith principle[15].

Appendix

Proof of Proposition 1

Since each contract makes nonnegative profit at a market equilibrium, Proposition 1 will be established if we show that insurers make zero profit. Assume ***a contrario*** that insurers make positive profit at an equilibrium $C_\ell = (k_\ell, x_\ell), C_h = (k_h, x_h), s_h, s_\ell, p.$ Let $C'_\ell = (k'_\ell, x'_\ell), C'_h = (k'_h, x'_h)$ be another pair of contracts offered in addition to $C_\ell, C_h.$ Let $\tilde{p}$ and $\tilde{p}'$ be respectively the auditing probability for C_ℓ and C'_ℓ in this deviation from equilibrium. We will show that C'_ℓ, C'_h can be chosen in such a way that it leads to an increase in profit (at all equilibria of the corresponding continuation subgame).

Standard arguments of the Rothschild-Stiglitz model show that such profit increasing contracts exist when C_ℓ, C_h is weakly incentive compatible (in the sense that h-types and ℓ-types have opposite weak preferences between C_ℓ and C_h if $p = 0$) and also when we have a pooling equilibrium where both types choose the same contract[16]. The case where C_ℓ, C_h is not weakly incentive compatible and both types choose different contracts and $p > 0$, requires more attention. At such an equilibrium, ℓ-types strongly prefer C_ℓ to C_h which implies $s_\ell = 1$. We also have $0 < s_h < 1$ because $p > 0$ implies $s_h < 1$ and the equilibrium is pooling when $s_h = 0$. Hence h-types randomize between C_h and C_ℓ and p makes them indifferent between both contracts. We then have

$$\begin{aligned} & (1-\pi_h)U(W_N - k_h) + \pi_h U(W_A + x_h) \\ = \; & (1-\pi_h)U(W_N - k_\ell) + \pi_h[(1-p)U(W_A + x_\ell) + pU(W_A)] \end{aligned}$$

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[15] On game theoretic models of markets with adverse selection, see Engers and Fernandez (1987) and Hellwig (1987).

[16] In such cases, the fact that there may be auditing for C_ℓ does not affect the arguments of Rothschild and Stiglitz (1976) because Π_ℓ - given by (7) - only depends on s_h and s_ℓ : it does not depend on p.

Let $k^0_\ell = k_\ell - \varepsilon; x^0_\ell = x_\ell; k^0_h = k_h - \varepsilon$ and $x^0_h = x_h$; with $\varepsilon > 0$. Let $p^*(\varepsilon)$ be the auditing probability that makes h-types indifferent between C^0_h and C^0_ℓ, with $p^*(\varepsilon) \quad p \quad (0;1)$ when $\varepsilon \quad 0$.

We know from Lemma 1 that $qx_\ell = c$ when $p > 0$. We here have $q < 1$ (because $s_\ell = 1$) which implies $x_\ell > c$. Using (3) and $qx_\ell = c$ gives

$$s_h = 1 - \frac{c(1-\lambda)\pi_\ell}{\lambda\pi_h(x_\ell - c)} \tag{12}$$

For ε small enough, the only continuation equilibrium following the deviation C^0_h, C^0_ℓ is such that h-types randomize between C^0_h and C^0_ℓ (they choose C^0_ℓ with probability s_h and C^0_h with probability $1-s_h$), ℓ-types choose C^0_ℓ and $\bar{p}^0 = p^*(\varepsilon)$[17] and nobody chooses C_ℓ or C_h[18]. Hence, following the deviation, all ℓ-types choose C^0_ℓ and h-types randomize between C^0_h and $C^0_\ell$with probabilities s_h and $1 - s_h$, which gives an increase in profit if ε is small enough.

Proof of Proposition 4

Let $C_h = C^*_h$ and C_ℓ be contracts offered at a semiseparating equilibrium. We have $s_\ell = 1$ and $s_h \quad (0;1)$: h-types are indifferent between C^*_h and C_ℓ which implies

$$\begin{aligned} U(W_N - \pi_h A) = \; & (1-\pi_h)U(W_N - k_\ell) \\ & + \pi_h\left[(1-p)U(W_A + x_\ell) + pU(W_A)\right] \end{aligned}$$

Hence

$$p = \frac{(1-\pi_h)U(W_N - k_\ell) + \pi_h U(W_A + x_\ell) - U(W_N - \pi_h A)}{\pi_h\left[U(W_A + x_\ell) - U(W_A)\right]} \tag{13}$$

Assume $p = 0$. Then C_ℓ is indifferent to C^*_h for h-types. There is a contract C^0_ℓ near C_ℓ which ℓ-types prefer to C_ℓ and the h-types prefer C^*_h to

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[17] Indeed, let s^0_h be the probability for a *h*-types to choose C^0_ℓ. The *h*-types' equilibrium contract choice is $s^0_h = 1$ if $p^0 < p^*(\ )$ $s^0_h = 0$ if $p^0 > p^*(\ )$ and $s^0_h \quad [0\ 1]$ if $p^0 > p^*(\ )$. The insurer who offers C^0_ℓ choose $p^0 = 1$ if $s^0_h > 1 - s_h$, $p^0 = 0$ if $s^0_h < 1 - s_h$ and $p^0 \quad [0\ 1]$ if $s^0_h = 1 - s_h$ Hence the unique equilibrium in the continuation subgame following the offer of C^0_h C^0_ℓ is $\bar{p}^0 = p^*(\ )$ and $s^0_h = 1 - s_h$.

[18] Note that there cannot exist a continuation equilibrium where *h*-types choose C_ℓ with positive probability. Indeed in such a case, *h*-types would be the only buyers of C_ℓ. Since $x_\ell > c$, the optimal verification strategy would be $\bar{p} = 1$ and C_ℓ could not be an optimal choice for *h*-types, hence a contradiction. Randomizing between C^0_h and C^0_ℓ only is actually an optimal strategy of *h*-types if $\bar{p} = 1$ Such an audit probability is an optimal strategy of the insurers who offer C_ℓ - when C^0_h C^0_ℓ are offered in deviation - if choosing C_ℓ were interpreted by insurers as a strategy played by *h*-types only, which is possible as an out of equilibrium belief.

C'_ℓ. Thus C'_ℓ is profitable[19]. Hence $p = 0$ cannot be an equilibrium verification strategy. We thus have $0 < p < 1$ which gives (10). Using (7),(10) and $s_\ell = 0$ allows us to write $\Pi_\ell = 0$ as $k_\ell = \phi(x_\ell, c)$ where $\phi(x_\ell, c)$ is given by (8).

Let us show that $C_\ell = (k_\ell, x_\ell)$ maximizes the expected utility of ℓ-types subject to $k_\ell = \phi(x_\ell, c)$. ***A contrario,*** assume that there exists $\tilde{C}_\ell = (\tilde{k}_\ell, \tilde{x}_\ell)$ such that $\tilde{k}_\ell = \phi(\tilde{x}_\ell, c)$ and that ℓ-types prefer $\tilde{C}_\ell$ to C_ℓ. Let $C'_\ell = (k'_\ell, x'_\ell) = \varepsilon \tilde{C}_\ell + (1 - \varepsilon) C_\ell$, with $0 < \varepsilon < 1$. Assume that C'_ℓ is offered in addition to C_h^*, C_ℓ. Let $\tilde{p}$ and $\tilde{p}'$ be the auditing probability for C_ℓ and C'_ℓ in this deviation. Concavity of U implies that ℓ-types prefer C'_ℓ to C_ℓ. For ε small enough, h-types prefer C'_ℓ to C_h^* when there is no auditing. In a continuation equilibrium after C'_ℓ is offered, h-types do not choose C_ℓ anymore because we would have $\tilde{p} = 1$ in that case. Hence, the only possible continuation equilibrium is such that ℓ-types choose C'_ℓ and h-types randomize between C_h^* and C'_ℓ. Then $\tilde{p}'$ makes h-types indifferent between C_h^* and C'_ℓ, i.e. it satisfies (13) where (k'_ℓ, x'_ℓ) is substituted to (k_ℓ, x_ℓ) and h-types choose C'_ℓ with a probability given by (10) where x'_ℓ is substituted to x_ℓ. Note that this probability is less than 1 for ε small enough. Note also that $\tilde{p} = 1$ is an optimal strategy of the insurers if choosing C_ℓ were interpreted as a strategy played by h-types only, which is possible as an out of equilibrium belief. Not choosing C_ℓ is then an optimal strategy of h-types in this deviation.

Let Π'_ℓ be the expected profit drawn from C'_ℓ. We have

$$\begin{aligned} \Pi'_\ell &= k'_\ell - \phi(x'_\ell, c) \\ &= \varepsilon \tilde{k}_\ell + (1 - \varepsilon) k_\ell - \phi(\varepsilon \tilde{x}_\ell + (1 - \varepsilon) x_\ell, c) \end{aligned}$$

Using $\partial^2 \phi(x, c)/\partial x^2 > 0$ allows us to write

$$\Pi'_\ell > \varepsilon[\tilde{k}_\ell - \phi(\tilde{x}_\ell, c)] + (1 - \varepsilon)[k_\ell - \phi(x_\ell, c)] = 0$$

which shows that the deviation is profitable, thereby contradicting the definition of an equilibrium.

Since $\hat{C}_\ell$ maximizes $(1 - \pi_\ell)U(W_N - k_\ell) + \pi_h U(W_A + x_\ell)$ with respect to x_ℓ, k_ℓ subject to $k_\ell \geq \phi(x_\ell, c)$, we have

$$\frac{\pi_\ell U'(W_A + \hat{x}_\ell)}{(1 - \pi_\ell)U'(W_N - \hat{k}_\ell)} = \frac{\partial \phi}{\partial x}(\hat{x}_\ell, c) < \frac{\pi_\ell}{1 - \pi_\ell}$$

which implies $U'(W_A + \hat{x}_\ell) < U'(W_N - \hat{k}_\ell)$. Using $U'' < 0$ then gives $W_A + \hat{x}_\ell > W_N - \hat{k}_\ell$ or $\hat{x}_\ell + \hat{k}_\ell > W_N - W_A = A$.

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[19] If C'_ℓ is offered, h-types either choose C_h^* or possibly C_ℓ if not auditing remains an optimal verification strategy (which is the case if $x_\ell < c$).

Lemma 2 ***When C_h^* and C_ℓ^{**} are offered, there exists a profitable incentive compatible deviation $C_h'; C_\ell'$ (i.e. a deviation where h-types choose C_h', ℓ-types choose C_ℓ' and there is no auditing in a continuation equilibrium) if and only if $\lambda < \lambda^*$; with $\hat{\lambda} < \lambda^* < 1$***

Proof. Assume that C_h^* and C_ℓ^{**} are offered in the market. A deviation $(C_\ell'; C_h')$ is incentive compatible when :

$$\begin{aligned} & (1-\pi_h)U(W_N - k_h') + \pi_h U(W_A + x_h') \\ \geq \ & (1-\pi_h)U(W_N - k_\ell') + \pi_h U(W_A + x_\ell') \end{aligned} \tag{14}$$

$$(1-\pi_h)U(W_N - k_h') + \pi_h U(W_A + x_h') \geq U(W_N - \pi_h A) \tag{15}$$

$$\begin{aligned} & (1-\pi_\ell)U(W_N - k_\ell') + \pi_\ell U(W_A + x_\ell') \\ \geq \ & 1-\pi_\ell)U(W_N - k_h') + \pi_\ell U(W_A + x_h') \end{aligned} \tag{16}$$

$$\begin{aligned} & (1-\pi_\ell)U(W_N - k_\ell') + \pi_\ell U(W_A + x_\ell') \\ \geq \ & (1-\pi_\ell)U(W_N - \pi_\ell A') + \pi_\ell U(W_A + (1-\pi_\ell)A') \end{aligned} \tag{17}$$

From (14) and (15), C_h' is a best choice for h-types when there is no auditing and from (16) and (17), C_ℓ' is a best choice for ℓ-types. When h-types choose C_h' and ℓ-types choose C_ℓ'; the agregate profit from $C_\ell'; C_h'$ is

$$\Pi' = (1-\lambda)\left[(1-\pi_\ell)k_\ell' - \pi_\ell x_\ell'\right] + \lambda\left[(1-\pi_h)k_h' - \pi_h x_h'\right] \tag{18}$$

Let $\mathbf{P_1}$ denote the problem where Π' given by (18) is maximized with respect to $k_\ell'; x_\ell'; k_h'; x_h'$ subject to conditions (14) to (17). Let $\Gamma_1(\lambda)$ be the value function of $\mathbf{P_1}$, i.e. the profit reached in $\mathbf{P_1}$ as a function of λ: There exists a profitable incentive compatible deviation $C_\ell'; C_h'$ if and only if $\Gamma_1(\lambda) > 0$:

Observe that we can restrict the set of feasible solutions in $\mathbf{P_1}$ by assuming that C_ℓ' does not provide overinsurance, i.e. $x_\ell' + k_\ell' \leq A$. Indeed when $x_\ell' + k_\ell' > A$, there exists another contract $C_\ell'' = (k_\ell''; x_\ell'')$ with $x_\ell'' + k_\ell'' \leq A$ such that C_ℓ' and C_ℓ'' are considered equivalent by ℓ-types and both contracts yield the same expected profit when chosen by a ℓ-type. Furthermore, h-types prefer C_ℓ' to C_ℓ''. Hence replacing C_ℓ' by C_ℓ'' relaxes the incentive compatibility contraint of the h-types, without change on profit.

Let $\mathbf{P_1'}$ be the relaxed problem obtained by deleting (16) in $\mathbf{P_1}$. We will verify hereafter that any optimal solution of $\mathbf{P_1'}$ satisfies (16) and thus that both problems have in fact the same optimal solutions. Any feasible solution of $\mathbf{P_1'}$ is weakly dominated by a solution where C_h' provides full insurance.

We may thus write $C_h^0 = (k_h^0; x_h^0) = (\pi_h^0 A; A - \pi_h^0 A)$ and (14) and (15) are then respectively rewritten as

$$U(W_N - \pi_h^0 A) \geq (1 - \pi_h) U(W_N - k_\ell^0) + \pi_h U(W_A + x_\ell^0) \tag{19}$$

and

$$\pi_h^0 \leq \pi_h \tag{20}$$

An optimal solution of $\mathbf{P}_1^0$ is obtained by maximizing

$$\Pi^0 = (1 - \lambda)[(1 - \pi_\ell)k_\ell^0 - \pi_\ell x_\ell^0] + \lambda(\pi_h^0 - \pi_h)A \tag{21}$$

with respect to $k_\ell^0; x_\ell^0; \pi_h^0$ subject to $x_\ell^0 + k_\ell^0 \leq A$ and (17), (19,(20). In problem $\mathbf{P}_1^0$; (17)and (19) are obviously binding at the optimum and $\pi_h^0 < \pi_h$ only if $\Pi^0 > 0$. Furthermore, ℓ-types prefer C_ℓ^0 to C_h^0 if $x_\ell^0 + k_\ell^0 < A$ and $C_\ell^0 = C_h^0$ if $x_\ell^0 + k_\ell^0 = A$. Hence (16) is satisfied at any optimal solution of $\mathbf{P}_1^0$. We deduce that $\mathbf{P}_1^0$ and $\mathbf{P}_1$ have the same optimal solution and $\Gamma_1(\lambda)$ is also the value function of $\mathbf{P}_1^0$.

$\Gamma_1(\lambda)$ is continuous over [0,1] and the envelope theorem in $\mathbf{P}_1^0$ gives

$$\Gamma_1^0(\lambda) = (\pi_h^0 - \pi_h)A - [(1 - \pi_\ell)k_\ell^0 - \pi_\ell x_\ell^0]$$

$(k_\ell^0; x_\ell^0; \pi_h^0) = (\pi_\ell A^0; (1 - \pi_\ell)A^0; \pi_h)$ is a feasible solution of $\mathbf{P}_1^0$. When this solution is optimal, we have $\Gamma_1(\lambda) = 0$: Otherwise, we have $\Gamma_1(\lambda) > 0$, $\pi_h^0 < \pi_h$ and $(1 - \pi_\ell)k_\ell^0 - \pi_\ell x_\ell^0 > 0$ which gives $\Gamma_1^0(\lambda) < 0$: Since $\Gamma_1(0) > 0$, there exists λ^* in $(0; 1]$ such that $\Gamma_1(\lambda) > 0$ if $0 < \lambda < \lambda^*$ and $\Gamma_1(\lambda) = 0$ if $\lambda^* \leq \lambda \leq 1$. When $\lambda < \hat{\lambda}$, there exists a solution to $\mathbf{P}_1$ such that $C_h^0 = C_\ell^0$ and $\Pi^0 > 0$; which implies $\hat{\lambda} < \lambda^*$.

It remains to show that $\lambda^* < 1$. Writing (17) as an equality gives $k_\ell^0 = f(x_\ell^0)$ with $f^0 > 0$ and $f^{00} < 0$: Let $\overline{x}_\ell$ given by $\overline{x}_\ell + f(\overline{x}_\ell) = A$: Condition $x_\ell^0 + k_\ell^0 \leq A$ is thus equivalent to $x_\ell^{**} \leq x_\ell^0 \leq \overline{x}_\ell$. Condition (19), now written as an equality, may be more compactly rewritten as $\pi_h^0 = g(k_\ell^0; x_\ell^0)$ with $g_1^0 > 0$ and $g_2^0 < 0$: Let $h(x_\ell^0) \equiv g(f(x_\ell^0); x_\ell^0)$. $h(x_\ell^0)$ is positive and decreasing over $[x_\ell^{**}; \overline{x}_\ell]$. $\mathbf{P}_1^0$ is rewritten as

$$\begin{aligned} \text{Maximize} \quad & (1 - \lambda)[(1 - \pi_\ell) f(x_\ell^0) - \pi_\ell x_\ell^0] \\ & + \lambda[h(x_\ell^0) - \pi_h]A \equiv V(x_\ell^0) \end{aligned}$$

with respect to x_ℓ^0 in $[x_\ell^{**}; \overline{x}_\ell]$. Using $f^{00} < 0$ gives

$$V^0(x_\ell^0) < \lambda \left[\frac{1 - \lambda}{\lambda} ((1 - \pi_\ell) f^0(x_\ell^{**}) - \pi_\ell) + \overline{h}^0 A \right] \quad \text{if} \;\; x_\ell^{**} < x_\ell^0 < \overline{x}_\ell$$

where $\overline{h'} = \mathrm{Sup}\{h'(x'_\ell); x^{**}_\ell \le x'_\ell \le \bar{x}_\ell\} < 0$. Hence $V'(x'_\ell) < 0$ for all x'_ℓ in $[x^{**}_\ell, \bar{x}_\ell]$ if $\lambda > \bar{\lambda}$ with

$$\bar{\lambda} \equiv \frac{(1-\pi_\ell)f'(x^{**}_\ell) - \pi_\ell}{(1-\pi_\ell)f'(x^{**}_\ell) - \pi_\ell - \overline{h'}A} \quad (0,1).$$

Thus $x'_\ell = x^{**}_\ell, k'_\ell = k^{**}_\ell, \pi'_h = \pi_h$ is an optimal solution to $\mathbf{P}'_1$ when $\bar{\lambda} < \lambda < 1$. Hence $\Gamma_1(\lambda) = 0$ if $\bar{\lambda} < \lambda < 1$, which implies $\lambda^* < 1$. ■

Proof of Proposition 5

Let $(C_h, C_\ell, s_h, s_\ell, p)$ be a market equilibrium . We know from Propositions 2 and 3 that $C_h = C^*_h$, and $s_\ell = 1$. Assume that $s_h = 1$. Then Lemma 1 gives $p = 0$. Hence $C_\ell = C^{**}_\ell$ for otherwise either C_ℓ is chosen by h-types (i.e. $s_h < 1$) or a profitable deviation C'_ℓ exists which attracts ℓ-types. Thus, the only candidate for a separating equilibrium is the Rothschild-Stiglitz pair of contracts $C_h = C^*_h, C_\ell = C^{**}_\ell$ without type verification.

$(C_h, C_\ell, s_h, s_\ell, p) = (C^*_h, C^{**}_\ell, 1, 1, 0)$ is a market equilibrium if no profitable deviation (C'_h, C'_ℓ) exists, with $C'_h = (k'_h, x'_h)$ and $C'_\ell = (k'_\ell, x'_\ell)$. Let $\hat{p}$ and $\hat{p}'$ be respectively the auditing probability for C_ℓ and C'_ℓ in this deviation from equilibrium.

By Lemma 2 there exists a profitable incentive compatible deviation if $\lambda < \lambda^*$. Obviously, in such a case a ***strongly*** incentive compatible deviation C'_h, C'_ℓ also exists, i.e. a deviation where the ***only*** continuation equilibrium is such that ℓ-types choose C'_ℓ , h-types choose C'_h and $\hat{p}' = 0$. If $c < c^*$ there exists a profitable deviation C'_h, C'_ℓ with a unique continuation equilibrium where ℓ-types choose C'_ℓ, h-types randomize between C'_h and C'_ℓ and $0 < \hat{p}' < 1$. Hence $\lambda \ge \lambda^*$ and $c \ge c^*$ are necessary conditions for a separating equilibrium to exist.

Conversely, assume $\lambda \ge \lambda^*$ and $c \ge c^*$. Let us show that for all deviations C'_h, C'_ℓ, there exists a continuation equilibrium, with auditing probabilities $\hat{p}$ and $\hat{p}'$, that makes the deviation unprofitable. We may restrict attention to deviations that attract ℓ-types since it is a necessary condition for a deviation to be profitable.

Assume first that ℓ-types weakly prefer C'_ℓ to C'_h.

If h-types weakly prefer C'_h to C'_ℓ when $\hat{p}' = 0$, then there exists a continuation equilibrium where ℓ-types choose C'_ℓ, h-types choose C'_h and $\hat{p}' = 0$. By Lemma 2, such a deviation cannot be profitable when $\lambda \ge \lambda^*$.

If h-types (strongly) prefer C'_ℓ to C'_h when $\hat{p}' = 0$, then a continuation equilibrium is as follows. If

$$c[(1-\lambda)\pi_\ell + \lambda\pi_h] < \lambda\pi_h x'_\ell$$

then ℓ-types choose C^0_ℓ, h-types randomize between C^0_ℓ and C^0_h and $0 < \hat{p}^0 < 1$: Such a deviation cannot be profitable when $c \geq c^*$:

If

$$c[(1-\lambda)\pi_\ell + \lambda\pi_h] \geq \lambda\pi_h x^0_\ell$$

then ℓ-types and h-types choose C^0_ℓ and $\hat{p}^0 = 0$: Such a pooling deviation is not profitable when $\lambda \geq \hat{\lambda}$ and ***a fortiori*** it is not profitable when $\lambda \geq \lambda^*$.

Assume now that ℓ-types (strongly) prefer C^0_h to C^0_ℓ.

If h-types weakly prefer C^0_h to C^0_ℓ when $\hat{p}^0 = 0$; there is a continuation equilibrium where all individuals choose C^0_h. Once again, for this continuation equilibrium, the deviation cannot be profitable when $\lambda \geq \hat{\lambda}$ and ***a fortiori*** it is not profitable when $\lambda \geq \lambda^*$.

If h-types (strongly) prefer C^0_ℓ to C^0_h when $\hat{p}^0 = 0$, then a continuation equilibrium is as follows. If $c \leq x^0_\ell$, the continuation equilibrium is pooling : both types choose C^0_h and we have $\hat{p}^0 = 1$; choosing C^0_ℓ is interpreted as a strategy played by h-types only as an out of equilibrium belief. If $c > x^0_\ell$, the continuation equilibrium is incentive compatible : ℓ-types choose C^0_h, h-types choose C^0_ℓ and $\hat{p}^0 = 0$. Such continuation equilibria cannot be profitable when $\lambda \geq \lambda^*$.

Proof of Proposition 6

The candidate semiseparating equilibrium characterized in Proposition 4 is truly an equilibrium if there does not exist any profitable deviation $(C^0_h; C^0_\ell)$ and if s_h given by (10) is positive.

First, we will derive conditions under which there exists a profitable ***strongly*** incentive compatible deviation, that is a deviation where the only continuation equilibrium is such that h-types chooses C^0_h, ℓ-types choose C^0_ℓ and there is no auditing. $\hat{p}$ and $\hat{p}^0$ still denote the auditing probability for C_ℓ and C^0_ℓ in a continuation equilibrium following the deviation.

The incentive compatibility for ℓ-types is written as

$$(1-\pi_\ell)U(W_N - k^0_\ell) + \pi_\ell U(W_A + x^0_\ell) \geq \overline{U}(c) \tag{22}$$

Let $\mathbf{P_2}$ denote the problem where Π^0 given by (18) is maximized with respect to $k^0_\ell; x^0_\ell; k^0_h; x^0_h$ subject to conditions (14),(15),(16) and (22). Let $\Gamma_2(\lambda; c)$ be the value function of $\mathbf{P_2}$. Arguments similar to the ones used in the proof of Lemma 2 shows that we may restrict attention to the case where C^0_ℓ does not provide overinsurance and C^0_h is a full insurance contract. We write $(k^0_h; x^0_h) = (\pi^0_h A; A - \pi^0_h A)$. Hence $\mathbf{P_2}$ has the same optimal solution than problem $\mathbf{P^0_2}$ where Π^0 given by (21) is maximized with respect to $k^0_\ell; x^0_\ell; \pi^0_h$ subject to

$k^0_\ell + x^0_\ell \leq A$;(19), (20) and (22). There exists a profitable incentive compatible deviation if and only if $\Gamma_2(\lambda;c) > 0$: On can also easily check that a pooling deviation (i.e. a deviation where both types choose the same contract at a continuation equilibrium) cannot be profitable when $\Gamma_2(\lambda;c) \leq 0$:

When $\Gamma_2(\lambda;c) > 0$, there exists a profitable strongly incentive compatible deviation, i.e. a deviation where ℓ-types strongly prefer C^0_ℓ to $\widehat{C}_\ell; C^*_h$ and C^0_h and h-types strongly prefer C^0_h to $C^0_\ell; C^*_h$: There is no continuation equilibrium where h-types choose $\widehat{C}_\ell$ with positive probability, because $\widehat{p} = 1$ would be the equilibrium audit probability which would make the h-types' strategy suboptimal. Hence, after this deviation, the only continuation equilibrium is such that h-types choose C^0_h. Choosing C^0_h is actually an optimal strategy of h-types if choosing $\widehat{C}_\ell$ were interpreted by the insurers as a strategy played by h-types only, which is possible as an out of equilibrium belief in the continuation subgame that follows the deviation: this gives $\widehat{p} = 1$ and $\widehat{p}^0 = 0$ as auditing probability in this continuation equilibrium.

$\Gamma_2(\lambda;c)$ is continuous and such that $\partial\Gamma_2/\partial c > 0$: When $\Gamma_2(\lambda;c) > 0$, we have $(1-\pi_\ell)k^0_\ell - \pi_\ell x^0_\ell > 0$ at an optimal solution of $\mathbf{P}^0_2$, which gives $\partial\Gamma_2/\partial\lambda < 0$. Note also that $\overline{U}(0)$ is the expected utility reached by ℓ-types under full insurance at fair premium, which implies $\Gamma_2(\lambda;0) < 0$ for all $\lambda > 0$ and $\Gamma_2(0;0) = 0$. Finally, we have $\Gamma_2(\lambda;c^*) \equiv \Gamma_1(\lambda)$:

If $\lambda \geq \lambda^*$, then $\Gamma_1(\lambda) = 0$ and thus $\Gamma_2(\lambda;c^*) = 0$. We then have $\Gamma_2(\lambda;c) \leq 0$ if and only if $c \leq c^*$.

If $\lambda < \lambda^*$; then $\Gamma_1(\lambda) > 0$ and thus $\Gamma_2(\lambda;c^*) > 0$. In that case, using $\Gamma_2(\lambda;0) < 0$ and the continuity of $\Gamma_2(\lambda;c)$ shows that there exists $\tilde{c}(\lambda)$ in $(0;c^*)$ such that $\Gamma_2(\lambda;\tilde{c}(\lambda)) = 0$. Furthermore $\tilde{c}(\lambda^*) = c^*$, $\tilde{c}(0) = 0$ and $\tilde{c}'(\lambda) = -(\partial\Gamma_2/\partial\lambda)/(\partial\Gamma_2/\partial c) > 0$: We then have $\Gamma_2(\lambda;c) \leq 0$ if and only if $c \leq \tilde{c}(\lambda)$.

Hence $\Gamma_2(\lambda;c) > 0$ is equivalent to $c > \tilde{c}(\lambda); \lambda < \lambda^*$ or $c > c^*; \lambda \geq \lambda^*$ and a profitable deviation exists in such a case.

Conversely, assume $\Gamma_2(\lambda;c) \leq 0$ i.e. $c \leq \tilde{c}(\lambda); \lambda < \lambda^*$ or $c \leq c^*; \lambda \geq \lambda^*$. Firstly, we will show that s_h given by (10) is positive. Using (7), $s_\ell = 0$ and $\Pi_\ell = 0$ gives

$$\frac{\widehat{k}_\ell}{\widehat{x}_\ell} = \frac{\lambda(1-s_h)\pi_h + (1-\lambda)\pi_\ell}{\lambda(1-s_h)(1-\pi_h) + (1-\lambda)(1-\pi_\ell)}$$

and we have $s_h > 0$ if

$$\frac{\widehat{k}_\ell}{\widehat{x}_\ell} < \frac{\lambda\pi_h + (1-\lambda)\pi_\ell}{\lambda(1-\pi_h) + (1-\lambda)(1-\pi_\ell)} \tag{23}$$

Let $C'_\ell = (k'_\ell; x'_\ell)$ with $x'_\ell + k'_\ell < A; k'_\ell/x'_\ell > \hat{k}_\ell/\hat{x}_\ell$ such that C'_ℓ and $\hat{C}_\ell$ are considered equivalent by ℓ-types and both contracts yield the same expected profit when chosen by a ℓ-type. Let π'_h given by

$$U(W_N - \pi'_h A) = (1 - \pi_h)U(W_N - k'_\ell) + \pi_h U(W_A + x'_\ell) \tag{24}$$

with $\pi'_h < \pi_h$. Using (24) and $U'' < 0$ gives

$$(\pi_h - \pi'_h)A > (1 - \pi_h)k'_\ell - \pi_h x'_\ell \tag{25}$$

$k'_\ell; x'_\ell; \pi'_h$ is a feasible solution to $\mathbf{P}'_2$: $\Gamma_2(\lambda; c) \le 0$ then implies

$$(1 - \lambda)[(1 - \pi_\ell)k'_\ell - \pi_\ell x'_\ell] + \lambda(\pi_h - \pi'_h)A \le 0 \tag{26}$$

(25) and (26) then give

$$(1 - \lambda)[(1 - \pi_\ell)k'_\ell - \pi_\ell x'_\ell] + \lambda[(1 - \pi_h)k'_\ell - \pi_h x'_\ell] < 0 \tag{27}$$

Using (27) and $k'_\ell/x'_\ell > \hat{k}_\ell/\hat{x}_\ell$ shows that (23) is satified, which establishes that $s_h > 0$.

It remains to show that for all deviations $C'_h; C'_\ell$, there exists a continuation equilibrium, with auditing probabilities $\hat{p}$ and $\hat{p}'$, that makes the deviation unprofitable. This part of the proof is a simple adaptation of the proof of Proposition 5. It is therefore omitted for the sake of brevity.

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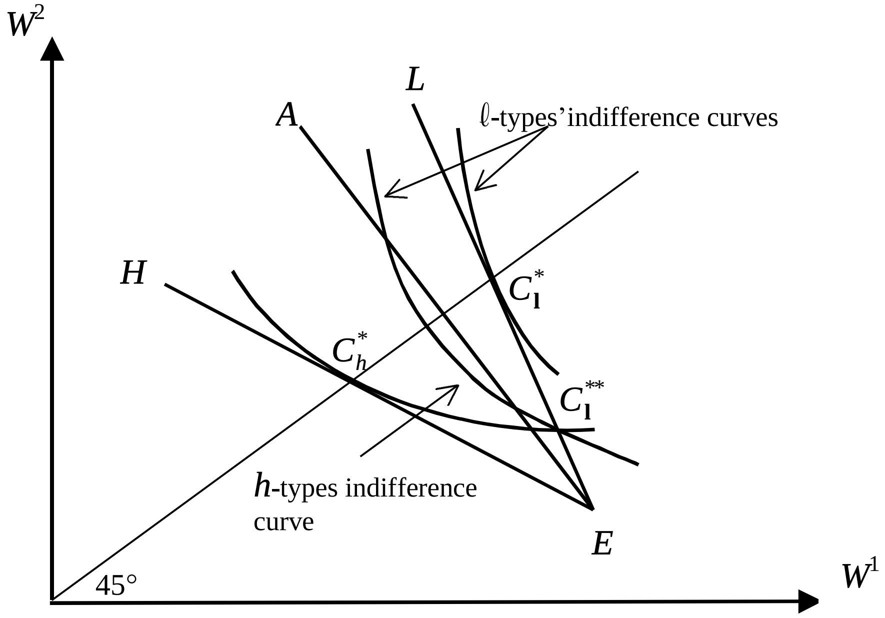

W^2
L
A
ℓ-types' indifference curves
H
C^*_ℓ
C^*_h
C^{**}_ℓ
h-types indifference curve
E
W^1
45°

Figure 1

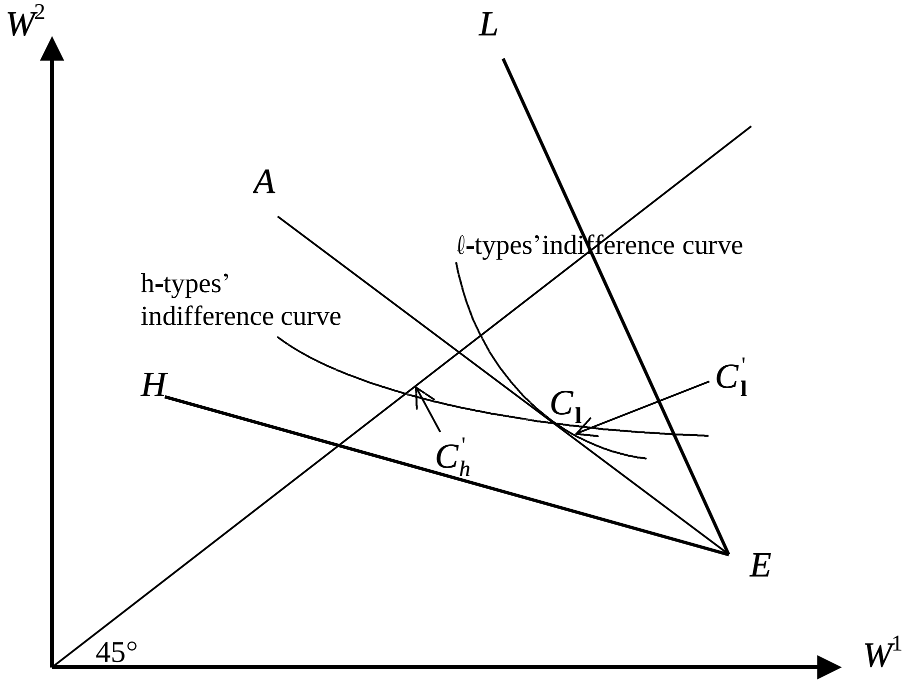
W^2
L
A
ℓ-types' indifference curve
h-types' indifference curve
H
$C_{\mathbf{l}}$
$C'_{\mathbf{l}}$
C'_h
E
45°
W^1

Figure 2

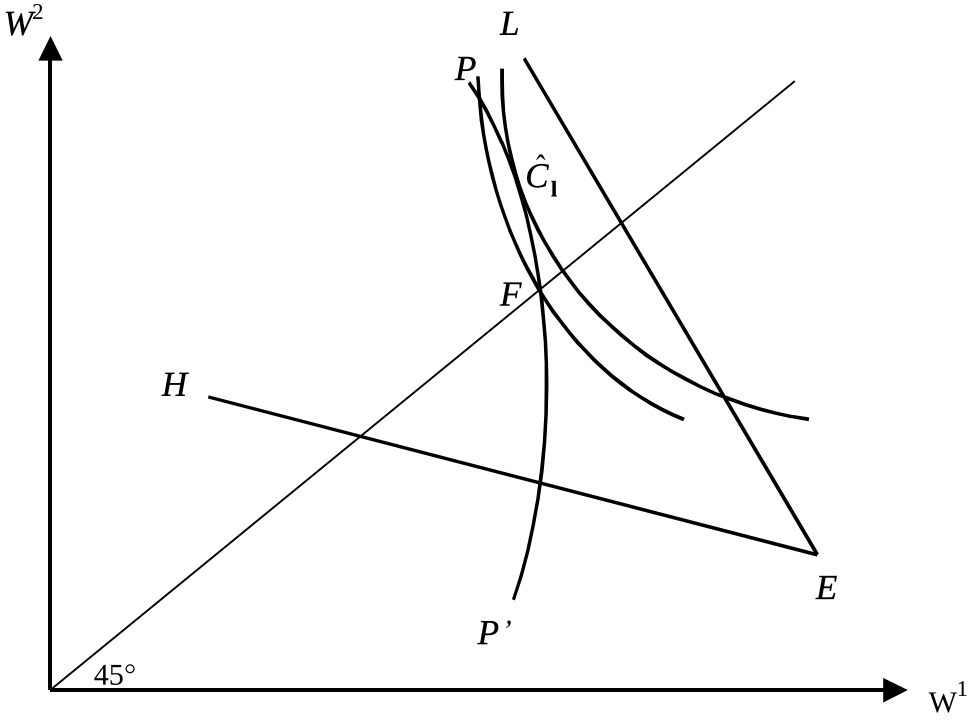

Figure 3

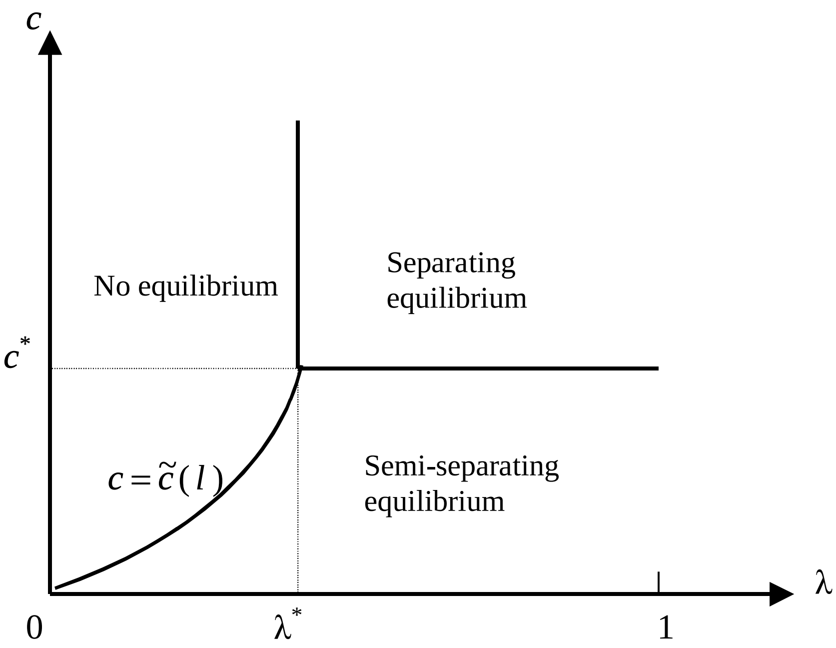
c
No equilibrium
Separating
equilibrium
c^*
$c = \tilde{c}(l)$
Semi-separating
equilibrium
λ
0
λ^*
1

Figure 4