Regulating an agent with different beliefs[1]

François Salanié[2] and Nicolas Treich[3]

February 18, 2002

[1]Preliminary.
[2]LEERNA, INRA and Université de Toulouse. salanie@toulouse.inra.fr
[3]LEERNA, INRA and Université de Toulouse. ntreich@toulouse.inra.fr

Abstract

There is some evidence that agents misperceive risks, such as lethal or environmental risks. Hence their behavior is based on beliefs which may differ from the 'objective' beliefs used by a regulator. The optimal regulation then depends on this difference in beliefs. We set up a general framework and study this policy change. It turns out that in many simple cases, the policy change depends on the absolute 'distance' between beliefs, and not on whether agents over-estimate or under-estimate risks. We characterize the necessary and sufficient condition for 'more distant' beliefs to always reduce the regulator's decision. We relate that condition to various problems, including the question of decision-making when some information will be made available in the future.

1 Introduction

1.1 The motivation

There is some evidence that the beliefs of people are biased. A famous example is the bias in lethal risks perception. Individuals systematically overestimate the rare causes of death such as cataclysmic storms or plane crashes and underestimate more common causes of death like cancers or automobile accidents (Lichtenstein et al., 1978).[1] Typically, individuals' beliefs on hazard risks and on environmental problems differ from those of experts (Slovic, 1986, Viscusi, 1998).

This paper is interested in the implications of this observation for risk regulatory policies: Should governments be concerned with the risks that people *perceive* they face? And, if yes, how should governments' policies account for public misperceptions?

Existing economic theory has not really answered these sort of questions: What weight should be accorded in social choices to individuals' erroneous beliefs? How will agents with different prior beliefs interact? This paper examines the effect of relaxing the common prior belief assumption within a model of regulation with two agents, a 'benevolent' government and an 'irrational' individual.

1.2 Related policy debate

The following problem has been introduced by Portney (1992, p. 131). It is called 'Trouble in Happyville':

You are Director of Environmental Protection in Happyville (...). The drinking water supply in Happyville is contaminated by a naturally occurring substance that each and every resident believes may be responsible for the above-average cancer rate observed there (...).

[1] The literature on public misperceptions and their causes is very well documented since the seminal paper by Tversky and Kahneman (1974). Individuals have difficulties with the mathematics of probability, they use heuristics or rules of thumbs that are useful but misleading. For instance, they are subject to 'availability heuristic'. People assess the risks of heart attack by recalling such occurences among one's acquaintances. They 'anchor' their estimates to easily retrievable events in memory such as sensational stories in the medias etc..

You have asked the top ten risk assessors in the world to test the contaminant for carcinogenicity (...). These risk assessors tell you that while one could never prove that the substance is harmless, they would each stake their professional reputations on its being so.

You have repeatedly and skillfully communicated this to the Happyville citizenry, but because of the deep-seated skepticism of all government officials, they remain completely unconvinced and truly frightened.

The mirror image of Happyville is Blissville (Viscusi, 2000). In Happyville, the risk is low but perceived as large. In Blissville, the risk is important but perceived as low. The question becomes: You are the Director of Environmental Protection both in Happyville and Blissville, how do you allocate your cleanup efforts? To Viscusi, the choice is clear-cut. Efforts have to be spent in Blissville. If efforts are spent in Happyville, this is a 'statistical murder' since lives are sacrificed to focus instead on illusory fears. Viscusi (2000)'s view is probably shared by most economists.

In Policy Sciences, many sholars have argued that the choice is not so clear-cut (Pollack (1998) and references therein). If efforts are spent in Happyville, people who were worried feel protected, and so feel better. This contrasted view is particularly apparent in Europe. European regulatory choices reflect more the differences in policy judgments and cultural values and are filtered through national institutions (Pollack, 1995). A recent report of the European Commission states for instance 'Decision-makers have to account of the fears generated by the perceptions and to put in place preventive measures to eliminate the risks' (CEC, 2000). Such a view is called the 'populist approach' to risk regulation by Hird (1994).

There are many arguments against a 'populist' approach to risk management. Using data on American health risk programs, Viscusi (1998) has investigated the failures of the regulatory policies based on a 'populist' approach as opposed to a cost-benefit or say a 'rational' approach. The cost has been millions of dollars for the U.S., or said differently, it has been thousands of American lives. This is what Breyer (1993) calls the 'vicious circle' of risk regulation: Individuals' misperceptions are embodied in government regulations. There are many reasons for that. The simplest reason is because politicians are subject to the same biases in beliefs as individuals. A complex reason is because politicians respond in some way to individuals' preferences and biases. As Margolis (1996, p. 161) states 'If enough people feel worried about some risk, however remote and cautiously calculated, then it makes sense to say the the government ought to respond to that. How to respond is less clear'.

This moves the policy debate from a normative to a more positive question: 'How to respond?'. In Pollack (1998, p. 379)'s words, 'How do governments regulate risks when the perceptions of the public diverge from those of experts? (..) What role do risk analysis and cost-benefit analysis play?'. Pollack argues that those questions should deserve probably much more attention than they have received so far.

1.3 Our approach to the debate

Let now present our approach to this debate. An individual faces a risk $\widetilde{x}$, and believes that the distribution of the risk is q. Given a regulatory environment a (road safety, cigarette prices..), he makes a choice b (driving speed, smoking...). His objective is to choose b to get the best expected utility

$$E_qU(\widetilde{x}, a, b),$$

which yields a decision $b(a, q)$.

Importantly, this decision made by the individual is based on the subjective probability or *perceived* risk q for $\widetilde{x}$, not on the objective probability or the *actual* risk p. The decision $b(a, q)$ thus maximizes perceived expected utility E_qU, not actual expected utility E_pU.

Let now address the government regulatory policy. This policy is based on the choice of a. The objective of the government is to maximize the *actual* expected utility of the individual E_pU, i.e. the expected utility based on the objective risk, p. The government chooses a to get the best

$$E_pU(\widetilde{x}, a, b(a, q)),$$

which yields the decision $a(p, q)$. Importantly, the government thus accounts for the irrational individual's response, i.e. the response based on the perceived risk q. This is the sort of 'second best' choice that is selected by the government.

This framework thus captures a complex channel for why the individual's perception q affect the efficacy of regulatory choices. This channel is related to the anticipation of the irrational response of individuals, i.e. the response based on q, not on p.[2]

[2] This means that in Happyville there are complex spill-over effects (Viscusi, 1998). A

More generally, our approach allows us to consider three polar cases. The regulator may select:

a(p,q): the 'second-best' policy,
a(p,p): the 'rationalist' policy,
a(q,q): the 'populist' policy.

The 'rationalist' approach is clearly inefficient because it does not anticipate correctly the agent's reactions. The 'populist' approach is intuitively inefficient because it does not make use of the regulator's information. The 'second-best' approach is illustrated in the following example.

1.4 An example: Regulation in Happyville

Let us come back to the choice faced by the Director of Environmental Protection in Happyville. Let

$$U(x, a, b) = u(b) - (1 - a)b\delta x - c(a),$$

where

$u(.)$	is the individual's utility from drinking water,
b	is water consumption,
a	is cleanup effort, $0 \leq a \leq 1$,
δ	is the desutility from getting a cancer,
x	is the dose-response risk of carcinogenicity,
$c(.)$	is the cleanup cost function.

Assume very simplistic functional forms

$$\begin{aligned} u(b) &= -\frac{(1-b)^2}{2}, 0 \leq b \leq 1, \\ \delta &= 1, \\ c(a) &= \frac{a^2}{2}. \end{aligned}$$

good example of spill-over effect is related to the influence of automobile seatbelts in the United States in the late 60s. Since seatbelts reduced the risk of injury to the driver, an associated effect has been to make drivers driving fast, offsetting the benefits of the safety regulatory policy (Peltzman, 1975). Lowering speed limit on major highways decreases the average driving speed but increases the incentive to drive on back roads (Viscusi, 1998).

Assume also that the risk is binary with an objective probability p of $x = 1$, 0 otherwise.[3]

Given these assumptions, the agent simply chooses b to maximize

$$E_q U(a, b, \widetilde{x}) = -\frac{(1-b)^2}{2} - (1-a)qb - \frac{a^2}{2},$$

so that we get

$$b(a, q) = 1 - (1-a)q.$$

According to the intuition, optimal water consumption b is decreasing in the perceived probability of getting a cancer q and increasing in the level of cleanup efforts a.

The regulator chooses a to maximize

$$E_p U(\widetilde{x}, a, b(a, q)) = -\frac{((1-a)q)^2}{2} - (1-a)p(1-(1-a)q) - \frac{a^2}{2}, \quad (1)$$

so that we get

$$a(p, q) = \frac{p - 2pq + q^2}{1 - 2pq + q^2} \in [0, 1]. \quad (2)$$

We are now in a position to examine the effect of individual beliefs on the regulator decision. This effect is represented on figure 1. This figure represents cleanup efforts as a function of individual's misperceptions q.

Let us first examine the 'rationalist' regulator decision $a(p, p)$. This decision does not internalize individual's misperceptions, so it is a straight line on the figure. The 'rationalist' decision is insensitive to public beliefs.

Then, turn to the opposite case, the 'populist' decision which is equal to

$$a(q, q) = \frac{q}{1+q}.$$

This decision is too sensitive to public beliefs. In Happyville, i.e. in the city where the perceived risk is large, $q > p$, cleanup efforts are high. In Blissville, where the perceived risk is low, $q < p$, cleanup efforts are low.

Finally, let us turn to the most sophisticated decision, that is the 'second best' decision. From equation (2), decision $a(p, q)$ is decreasing in q then increasing in q. Importantly, this function takes a minimum at $q = p$. Thus we have $a(p, q) \geq a(p, p)$. This shows that the optimal decision is always larger than the 'rationalist' decision. Why is that?

[3]We make a slight abuse of notation here: the probability vector p reduces to a scalar.

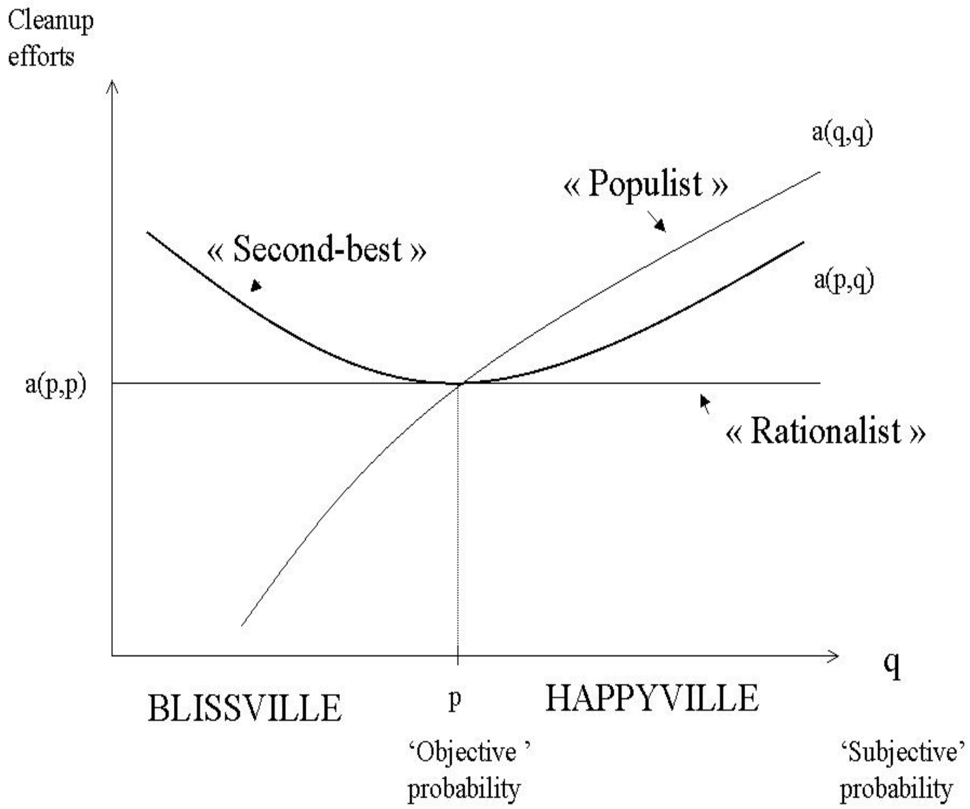

Figure 1: Cleanup efforts as a function of beliefs

In Happyville, individuals are pessimistic, they do not consume water enough. Cleaning water thus will make the risk lower. Happyville population will react to that change in risk. Hence, cleanup efforts gives an incentive for the population to consume more water, which is a source of welfare in Happyville where people overestimated the risk. In Blissville, the reason for why cleanup efforts increase is different. People are optimistic and consume too much water. Risk-exposure to cancer is thus too large in Blissville. Hence, cleaning water simply reduces risk-exposure.[4]

Finally, note that the difference between the 'second-best' policy $a(p,q)$ and the 'rationalist' policy $a(p,p)$ increases as the absolute value $|p-q|$ increases. Moreover, they increase exactly at the same rate. Indeed replace q by $p+u$ in (2) to get

$$a(p,p+u) = \frac{p(1-p)+u^2}{(1+p)(1-p)+u^2}$$

so that the value of a is independent of the sign of u. This means that cleanup efforts are the same in Blissville and Happyville.[5] Hence, an important lesson from that example is that the *difference* between the public and the regulator beliefs is more important than the direction of the misperception. In other words, it is not so much important for the regulator to know whether he is in Blissville or Happyville. What is important is to know 'how large' is the misperception.

To summarize: because the population's response is 'irrational', regulation may depart strongly from a myopic Cost-Benefit Analysis. Yet, this example has shown that this departure displays several 'regularity' properties. For instance, policy change depends on the absolute 'distance' between beliefs, not on whether agents over-estimate or under-estimate risks. This raises the question of the effect of different beliefs on regulatory policies in general.

The next Section introduces a general framework for studying the impact of distant beliefs. Section 3 offers a simple necessary and sufficient condition for distant beliefs to reduce the regulatory effort, and applies this condition

[4]This interpretation suggests that this result is model-dependent. We will precisely examine this question in the paper.

[5]The decisions are the same in the sense that $a(p,q)$ is symmetric around p. This symmetry is due to the selection of the parameters. For a different set of parameters, the symmetry is lost, though the message remains.

to the Happyville example. Section 4 derives an equivalent condition, based only on the primitives of the model. Section 5 discusses the relationships with other problems, in particular the question of decision-making when some information is awaited for in the future. Section 6 concludes.

2 The framework

The simplest manner to introduce our framework is the following. A regulator chooses a regulatory effort a. An agent reacts to a and chooses a decision b. These choices are performed under uncertainty on the true state of nature $x \in X$. The Von Neumann-Morgenstern preferences of the agent are given by the utility function $U(x, a, b)$. Because the regulator is only interested in the agent's welfare, he shares the same preferences.[6]

We assume that x takes a finite number of values, and by a slight abuse of notation we also denote this number by X; so that $X = \{x_1, .., x_X\}$. Decision a is a real number. Decision b is a real vector of dimension $N \geq 1$. Also, U is three-times continuously differentiable with respect to $(a,\, b)$.

Suppose first that the regulator and the agent have the same beliefs p, defined in the usual manner[7]:

$$\forall\ x \quad p(x) > 0 \qquad \sum_{x \in X} p(x) = 1.$$

Once a is chosen, the agent chooses b to maximize

$$\sum_{x \in X} p(x) U(x, a, b). \tag{3}$$

We assume that for any admissible p and a, this criterion is strictly quasi-concave in b. Consequently define $b(a, p)$ as the unique decision b which maximizes this criterion, and $j(a, p)$ as the value of the program:

$$j(a, p) = \max_b \sum_{x \in X} p(x) U(x, a, b) = \sum_{x \in X} p(x) U(x, a, b(a, p)). \tag{4}$$

[6] One could include in the regulator's preferences a cost for a: $V(x, a, b) = U(x, a, b) - c(a, x)$, without any change.

[7] In what follows we could suppose that weights belong to an open convex subset of all possible weights. The assumption that weights are strictly positive plays a role.

Hence, when both agents share the same beliefs, the best decision a must maximize $j(a, p)$. Notice that $j(a, p)$ is the maximum of functions which are linear in p. Hence $j(a, p)$ must be convex in p.

Now suppose that the weight q used by the agent differs from the weight p used by the regulator. Acting as a Stackelberg leader[8], the regulator should adjust his first-period decision consequently, by maximizing

$$\sum_{x \in X} p(x) U(x, a, b(a, q)). \tag{5}$$

over a. Our first objective in the following is to compare (5) to (4), i.e;, to study the impact of a change in the agent's beliefs. To do so, we need to introduce a measure for the difference in beliefs.

We will define a simple measure for that difference. Let introduce two weights p and q and two scalars $r, s \in [0, 1]$. Then $(1-r)p+rq$ and $(1-s)p+sq$ are also weights, and an increase in s makes the latter more distant from the former if $s > r$, and closer otherwise. Hence the absolute value $|s - r|$ is an index for the difference in beliefs. Define the regulator's expected payoff as

$$K(a, r, s) = \sum_{x \in X} [(1 - r)p(x) + rq(x)] U(x, a, b(a, (1 - s)p + sq)). \tag{6}$$

Here the regulator uses the weight $(1 - r)p + rq$, and the agent uses the weight $(1 - s)p + sq$. Such a definition still permits to consider our three polar cases:

- The 'second best' case, (r, s),
- The 'rationalist' case, (r, r),
- The 'populist' case, (s, s).

Notice that these linear forms appear quite naturally in many cases. For example, suppose that initially agents share the same beliefs, but an experiment is performed, giving additional information on the true state of nature. Nevertheless, there is an exogenous probability that the experiment has failed, and in that case its results are uninformative. Moreover there is

[8]We only study Nash-Perfect equilibria.

no way to tell whether the experiment has failed or not. If the regulator and the agent do not agree on the probability of failure, their revised beliefs take these linear forms.

In what follows, we shall make s vary in order to capture the impact of the difference in beliefs. We will say that beliefs are more *distant* if $|s-r|$ increases, r being given. Finally, the regulation problem $(r,\, s,\, p,\, q)$ is defined as the problem of maximizing (6) with respect to a. We assume that solutions to such problems always exist.

3 The impact of more distant beliefs

Our objective in this section is to investigate the effect of the difference in beliefs as defined above. We first examine this effect on the regulator's expected utility, then on the regulator's decisions. Note that in this section p and q are given, so that beliefs are restricted to belong to the straight interval $[p, q]$.

By definition of K and j, we have

$$K(a,r,r) = j(a,(1-r)p+rq) = \max_b \sum_x [(1-r)p(x)+rq(x)]U(x,a,b)$$

so that

$$\forall\ r,s \qquad K(a,r,s) \leq K(a,r,r). \tag{7}$$

This result states that the regulator's expected utility reaches its maximum when the agent and the regulator have the same beliefs, $s = r$. This raises the question of what happens when s moves progressively from r. This result is presented in the next proposition.[9]

Proposition 1 *The regulator's expected utility weakly increases with s for $s < r$, and weakly decreases with s for $s > r$.*

The meaning of that proposition is simple. More distant beliefs reduces the regulator's expected utility.

[9] The proof is given in appendix.

3.1 Equivalence results

Let us now turn to the effect of the difference in beliefs on the regulator's decision. Recall that the regulator maximizes the value function $K(a, r, s)$ as defined in (6). Hence the properties of the derivative K_a are essential here.

Suppose for example that we have

$$\forall\ a, r, s,\ \ K_a(a, r, s) \leq K_a(a, r, r). \tag{8}$$

Then it is clear that the optimal regulatory effort is reduced when different beliefs are introduced. Reciprocally, if this condition does not hold, then it is possible to build a case in which a is made higher with different beliefs[10]. Therefore (8) is equivalent to the fact that a difference in beliefs reduces a.

Now a quick look at the proof of Proposition 1 shows that the result is only based on inequalities (7) and the linearity of K. The same proof can then be applied to K_a, which is linear, if K_a verifies (8). Accordingly the Proposition that follows compares the regulator's decision in two polar cases, the 'second-best' decision based on (r, s) and the 'rationalist' decision, based on (r, r). Again the comparison extends to more distant beliefs.

Proposition 2 *The four following properties are equivalent:*
i) The regulator's decision decreases with more distant beliefs;
ii) $\forall\ a, r, s,\ \ K_a(a, r, s) \leq K_a(a, r, r)$*;*
iii) $\forall\ a, r, s,\ \ K_a(a, r, s)$ *increases with* s *when* $s < r$*, and decreases with* s *when* $s > r$*;*
iv) $K_a(a, 0, s)$ *decreases with* s*.*

This Proposition gathers necessary and sufficient conditions for signing the comparative statics analysis of more distant beliefs on the regulatory policy. The simplest condition is clearly iv), which asks to verify a simple property of the value function K.

Let us now give some examples of the usefulness of that proposition.

3.2 Examples

Example 1 *Prudence in Happyville*

[10] More precisely, one can use the freedom left by footnote 2, and choose a cost function $c(a)$ so that a is solution to problem (4), and a slightly higher a is solution to problem (5).

Let us now consider a more general Happyville population composed by a agents with a general utility function for drinking water u, assumed concave, and a general cleanup cost function $c(a)$. Now $b(a,s)$ is defined by

$$u'(b) = (1-a)s$$

so that K writes

$$K(a,r,s) = u(b(a,s)) - (1-a)b(a,s)r - c(a).$$

We then obtain

$$K_a(a,0,s) = u'(b(a,s))\frac{\partial b}{\partial a} = -s\frac{u'(b(a,s))}{u"(b(a,s))}$$
$$= -\frac{1}{1-a}\frac{u'(b(a,s))^2}{u"(b(a,s))}.$$

Since $b(a,s)$ is decreasing with s, the question that remains is whether

$$\frac{u'(b)^2}{u"(b)} \tag{9}$$

is increasing with b. Note that this property holds for

$$u(b) = -\frac{(1-b)^2}{2}.$$

But it does not need to hold for a general concave u. This shows that this is the modeling choice of a quadratic utility function that was at the root of the result displayed in the Introduction. In fact, it is easily shown that (9) is increasing with b if and only if the population displays a coefficient of prudence (Kimball, 1990) that is less than twice the coefficient of absolute risk-aversion. For an iso-elastic function (with constant relative risk-aversion α), the condition amounts to $\alpha < 1$; notice that for a logarithmic function, regulation is unaltered by the introduction of different beliefs.

Example 2 *Taxes in Happyville*

Suppose now that there is no cleanup technology available for the regulator. Yet, suppose that the regulator may set a tax a on water consumption. Preferences are given by

$$U(x,a,b) = u(b) - ab - bx.$$

The agent decision $b(a,s)$ is thus defined by

$$u'(b(a,s)) = a + s.$$

Hence

$$K(a,r,s) = u(b(a,s)) - ab(a,s) - rb(a,s)$$

so that

$$K_a(a,0,s) = [u'(b(a,s)) - a]\frac{\partial b}{\partial a} - b(a,s)$$
$$= \frac{u'(b) - a}{u"(b)} - b.$$

Since $b(a,s)$ is decreasing with s, there only remains to check whether this last expression is decreasing with b. This is true as soon as consumers are prudent ($u''' > 0$). Hence, more distant beliefs increases the optimal tax if and only if consumers are prudent.

4 Characterization results

The above examples were rather simple, since only the expected value of x mattered. In such a case, there is a natural ordering of beliefs, from the most optimistic to the most pessimistic. Now we have just seen that this ordering did not really matter for the choice of the optimal regulatory effort; what matters is the difference, not the direction. Does this insight extend to more general settings, in which no clear ordering of beliefs is available ?

It turns out that the answer is positive. To explain this, we have to redefine the objective function, by allowing p and q to vary: define

$$k(a,r,s;p,q) = \sum_{x \in X}[(1-r)p(x) + rq(x)]U(x,a,b(a,(1-s)p + sq)). \quad (10)$$

Then by linearity we get

$$k(a,r,s;p,q) = (1-r)k(a,0,s;p,q) + rk(a,1,s;p,q).$$

Now notice that

$$k(a,1,s;p,q) = k(a,0,1-s;q,p)$$

so that

$$k_s(a,r,s;p,q) = (1-r)k_s(a,0,s;p,q) - rk_s(a,0,1-s;p,q).$$

Finally recall that $k_s(a, s, s; p, q) = 0$, so that

$$(1 - s)k_s(a, 0, s; p, q) = sk_s(a, 0, 1 - s; p, q). \tag{11}$$

This shows that

Proposition 3 *For p, q given, suppose that the regulator with beliefs p reduces his regulatory effort when the agent's beliefs move from p to q. Then the regulator with beliefs q reduces his regulatory effort when the agent's beliefs move from q to p.*

Proof of Proposition 3: from Proposition 2, our assumption amounts to iv): $k_s(a, 0, s; p, q)$ decreases with a, for all s. From (11), we get that $k_s(a, 0, 1 - s; q, p)$ decreases with a. Apply Proposition 2 once more to get the result.■

This result supports a general message in this paper: what matters is not so much the direction in which beliefs are different, but simply the fact that beliefs are different. In the case when beliefs can be ordered from the most optimistic to the most pessimistic, this translates into a simple but unexpected property: one cannot find a setting in which the optimal regulatory effort increases when the agent becomes more optimistic, and decreases when the agent becomes more pessimistic.

We are thus led to look for settings in which more distant beliefs decrease the regulatory effort, whatever the direction. As this section shows, this turns out to be a strong property. To do so we need to introduce some notations. Denote U_a the derivative of U with respect to a; U_b the gradient of U with respect to b, and U_{ab} its derivative with respect to a; $U_{bb}(x, a, b)$ the Hessian matrix of U with respect to b. Finally define the hessian matrix of the objective in (4):

$$H(a, p) = \sum p(x)U_{bb}(x, a, b(a, p)).$$

Recall that by assumption H is negative definite, and thus invertible. Finally prime ($'$) stands for transposition.

Proposition 4 *The difference in beliefs always reduces a if and only if for any a, b, there exists a $N \times N$ matrix $M(a, b)$ and a N-vector $d(a, b)$ such that:*

i) for any x,

$$U_{ab}(x,a,b) + U_{bb}(x,a,b)d(a,b) = M(a,b)U_b(x,a,b).$$

ii) For any (p, q), if $b = b(a,q)$, then

$$(\sum_x p(x)U_b(x,a,b))'(M(a,b)+d_b(a,b))'[H(a,q)]^{-1}(\sum_x p(x)U_b(x,a,b)) \leq 0.$$

Let us comment this result. First, it asks to find M and d independent from x such that i) holds, for any x (this equality would not change if one wants the difference in beliefs to increase a). Such an equality is clearly non-generic, and requires specific functional forms for U.

In particular, if x represents beliefs as in the Happyville example, then U is linear with respect to x, and so is condition i). Hence i) reduces to two conditions specifying that both the constant and the x-factor are equal to zero (which gives $N(N+1)$ unknown for $2N$ equations).[11]

Second, choose $b = b(a,p)$ and sum i) over x; then one gets

$$\sum p(x)U_{ab}(x,a,b) + H(a,p)d(a,b) = M(a,b)[\sum p(x)U_b(x,a,b)] = 0$$

so that one must have

$$d(a,b(a,p)) = \frac{\partial b}{\partial a}(a,p).$$

Hence $d(a,b)$ characterizes how $b(a,p)$ varies with a. This also means that this partial derivative cannot depend directly upon p. This remark may help finding the vector d.

Third, ii) basically expresses that a matrix is positive[12] semi-definite (note that d_b is the differential of the vector d with respect to b, and is thus a matrix). In fact, a sufficient condition for ii) is that $M + d_b$ itself be positive semi-definite.

[11] Another example is studied in Ulph and Ulph[1997]. See also the example given in the Conclusion to this paper.

[12] Negative if one wants a to be increased by a difference in beliefs.

Fourth, these conditions only depend on the properties of U_b. This vindicates our view that any cost function $c(a, x)$ can be added to U, without any change.

Finally, these conditions slightly simplify in the case when b is unidimensional, as the following example illustrates.

Example 3 *Back in Happyville*

Let us generalize our Happyville model, by proceeding to a change of variables. Consider the case when

$$U(x, a, b) = v(a, b) - bx - c(a)$$

where, as before, a is water quality and $c(a)$ is the cleanup cost function. The change is that b is now an equivalent quantity, computed from the actual quantity consumed and water quality; a higher b indicates a higher exposure to the risk x. $v(a, b)$ is the surplus associated with the consumption of the equivalent quantity b, when water quality is a. Assume that $v_b > 0$, $v_{bb} < 0$, $v_{ab} > 0$; so that a better quality increases the equivalent consumption.

Let us apply Proposition 4. Note that U is linear with respect to x. Therefore condition i) splits into two conditions:

$$v_{ab} + dv_{bb} = mv_b \qquad 0 = -mx.$$

Therefore we must have $m = 0$ and $d = -v_{ab}/v_{bb}$. Condition ii) then reduces to $d_b \geq 0$. In other words, $\partial b/\partial a$ must be higher when b is higher. Since a higher b corresponds to a lower x, one would intuitively expect the opposite to be true; that is, an increase in water quality should have more effect on 'equivalent water' consumption when the damage x is high, compared to when the damage x is low.

Therefore we obtain that cleanup efforts are always increased by a difference in beliefs, if and only if the sensitivity of equivalent-water consumption to cleanup efforts is highest when the damage is highest.

5 Extensions

Being quite general, our framework allows for different interpretations[13]. More importantly, the framework can be used to solve completely a question first asked in Epstein[1980], about the impact on decision-making of the

[13] To give an example: consider a population of agents, characterized by $x \in X$, with preferences $U(x, a, b)$. Suppose that the government is utilitarist, and gives the weight

prospect of future information. The tools developed in this part turn out to also apply when the regulator faces a population of agents with different beliefs.

5.1 A link with the prospect of future information.

Let us consider in this part a quite different problem. Suppose that a decision-maker with *a priori* beliefs p and Von Neumann-Morgenstern preferences $U(x, a, b)$ sequentially chooses a and b. It is then easy to formalize the arrival of information, after decision a is taken but before decision b. Consider a random variable $\tilde{y}$ whose distribution conditional to x is known. Given some *a priori* beliefs p, a realization y of $\tilde{y}$ makes the decision-maker update his prior p into posterior beliefs q_y. Bayesian updating only requires that

$$p = E_y q_y \tag{12}$$

and the objective function of the decision-maker changes from (4) to

$$E_y \max_b \sum_{x \in X} q_y(x) U(x, a, b) = E_y j(a, q_y). \tag{13}$$

Now, from (12) and the convexity of j, it follows immediately that (13) is above $j(a, p)$: the prospect of information is always benefitial.

Similarly, consider the more general problem in which it is the informativeness (in the Blackwell[1951] sense) of future information which is learnt to be increased. A more precise information is defined as a random variable $\tilde{y}'$, such that any decision-maker prefers $\tilde{y}'$ to $\tilde{y}$. As is well-known, this is equivalent to the requirement that $\tilde{y}$ can be obtained from $\tilde{y}'$ by using a 'garbling machine', which adds a noise uncorrelated with the true state of nature. In other words, one can consider that the joint distribution of (x, y, y') is such that

$$Prob(y|y') = Prob(y|y', x).$$

Another useful result was obtained by Marshak-Myazawa[1968]: for any prior p, the distribution of posteriors $q_{y'}$ forms a mean-preserving spread of the

$p(x)$ to the agents of type x. Also, the government knows that the second-period decision will be taken by a different government, using the weights q. We are then back to the delegation problem, defined as the switch from (4) to (5). This difference reflects the impact of the change in governments.

distribution of posteriors q_y, in the (multi-dimensional) space of posteriors:

$$q_y = \sum_{y'} prob(y'|y) q_{y'}.$$

We then get

$$G(y,a) = E_y j(a, q_y) = E_y \sum_x q_y(x) U(x, a, b(a, q_y))$$

$$= \sum_{y,y'} Prob(y) prob(y'|y) \left[\sum_x q_{y'}(x) U(x, a, b(a, q_y)) \right]. \qquad (14)$$

We can also rewrite

$$G(y', a) = E_{y'} j(a, q_{y'}) = \sum_{y'} Prob(y') \sum_x q_{y'}(x) U(x, a, b(a, q_{y'}))$$

$$= \sum_{y,y'} Prob(y) Prob(y'|y) \left[\sum_x q_{y'}(x) U(x, a, b(a, q_{y'})) \right].$$

Comparing to (14), we see that the change from y' to a less informative y can be decomposed as a weighted sum of changes in the bracketted terms. Each of these changes is similar to a change from (4) to (5). Therefore our results apply; in fact, it turns out that there is an equivalence beween both class of problems:

Proposition 5 *The following properties are equivalent:*

i) More distant beliefs makes the optimal regulatory effort decrease.

ii) The prospect of more information makes the optimal regulatory effort increase.

Hence our problem shares a common structure with the question of decision-making under uncertainty. We can then use classical results, among which a well-known result due to Epstein[1980] and used in many subsequent papers:

Proposition 6 Epstein[1980] *The prospect of more information makes the optimal regulatory effort increase if and only if the derivative $j_a(a, p)$ is convex with respect to p.*

Note nevertheless that this last property is not easily verified, and a case-by-case study is often needed. Alternatively, one can iv) in Proposition 2 to deal with information problems. Indeed, verifying that K_a is decreasing with s may be easier than showing that j_a is convex in p. Finally, the last option is to check that the conditions in Proposition 4 apply.

5.2 Uncertain different beliefs

The above result is also useful to analyze wat happens in our regulator-agent framework, when the regulator faces a population of agents with different beliefs (or when the regulator is unsure of the agent's beliefs).

Suppose that there is a finite number of such possible beliefs, and denote a belief q_y, where y takes a finite number of values. Assume that x and y are independent; that is, the distribution of beliefs in the population does not convey any information on x. Hence the regulator knows that a proportion $prob(y)$ of the population has beliefs q_y. The expected payoff of the regulator is thus

$$\sum_y p(y)\left[\sum_x p(x)U(x,a,b(a,q_y))\right].$$

It is clear that such a criterion is built from the criterion

$$\sum_x p(x)U(x,a,b(a,p)) = \sum_y p(y)\left[\sum_x p(x)U(x,a,b(a,p))\right]$$

by applying a sery of changes in the bracketted terms. Each of these changes is similar to the change from (4) to (5). We then get:

Proposition 7 *The following properties are equivalent:*
i) More distant beliefs reduces the regulatory effort,
ii) Facing a population of agents with different beliefs reduces the regulatory effort.

Therefore all our results extend to the case when the regulator faces a population of agents.

6 Conclusion

This paper has introduced a model of regulation with two agents. The novelty has been to assume that the two agents display different prior beliefs. Although there is some empirical evidence that support that assumption, this is an unusual assumption in economics that is often considered as inconsistent.

In this case, the paper would reduce to a mathematical object. Can this object be useful in economics? The answer is yes. The idea is to reinterpret the model so that agents' beliefs are the same but their preferences differ. In fact, there are several 'classical' models where our mathematical object could be applied.[14] To understand how that could be done, a simple way is to consider the 'trendy' economic model of self-control.

Take the following three-periods consumption model

$$K(a, r, s) = u(a) + \beta u(b(a, s)) + \beta^2 r u(w - a - b(a, s)),$$

where $b(a, s)$ is defined by

$$u'(b(a, s)) = \beta s u'(w - a - b(a, s)).$$

This latter equation characterizes the optimal level of consumption of self-2 while the former characterizes the value function of self-1, given self-2 decision. Note that preferences between self-1 and self-2 differ in that framework. Indeed, self-1's discount factor between period 2 and period 3 is βr. Self-2's discount factor is βs. Hence the marginal rate of substitution between period 2 and 3 changes depending on whether this rate is computed from self-1 or self-2 perspective. This introduces a problem of time-inconsistency. Self-1 cannot perfectly control self-2 consumption decision.

In such a model, the most natural measure of lack of self-control is captured by the distance $|r - s|$. The difference between the marginal rates between period 2 and 3 increases with that distance. In this framework, our results state that self-1 expected utility decreases with the lack of self-control (Proposition 1), that self-1 consumption decreases or increases depending on the sign of the derivative $K_a(a, 0, s)$ in s (Proposition 2), and, finally, that this problem belongs to the set of problems that yield unambiguous results (Proposition 4). Thus a necessary and sufficient condition exists for signing the comparative statics analysis of less self-control.

This exact condition[15] and several extensions are presented in a companion paper (Salanié and Treich, 2002). A first insight from that paper is that this condition is such that it is perfectly plausible that the lack of self-control increases, and not decreases, self-1 savings. A more general insight is that the qualitative effect of self-control is independent of the structure of discount rates, i.e. it is the same no matter whether preferences are present-biased $r \leq s$ or future-biased $r \geq s$. In other words, self-1 does not care so much

[14]The Rotten-Kid Theorem is one example (Bergstrom, 1989). In this situation, the comparative statics analysis is about 'how much' the kid is rotten.

[15]The condition is the now well-known condition $P \geq 2A$.

about whether self-2 saves too much or too little from his viewpoint. He does care about 'how much' self-2 is different. These results convey quite different messages from the ones which are generally delivered in the economic literature on self-control.

References:

Bergstrom, T., 1989, A fresh look at the Rotten Kid Theorem, *Journal of Political Economy*, 97-5, 1138-1159.

Blackwell, D., 1951, Comparison of Experiments, in J. Neyman, ed. *Proceedings of the Second Berkeley Symposium on Mathematical Statistics and Probability*, Berkeley, Univesrsity of California Press, pp93-102.

Breyer, S. G., 1993, *Breaking the Vicious Circle: Toward Effective Risk Regulation*, Harvard University Press.

CEC (Commission of the European Communities), 2000, Communication from the commission on the Precautionary Principle, available at www.europa.eu.int.

Esptein, L.G., 1980, Decision-making and the temporal resolution of uncertainty, *International Economic Review*, 21, 269-284.

Gollier, C., 2001, 'The Economics of Risk and Time', MIT Press.

Gollier, C., B. Jullien and N. Treich, 2000, Scientific progress and irreversibility: An economic interpretation of the Precautionary Principle, *Journal of Public Economics*, 75, 229-53.

Hird, J., 1994, *Superfund: The Political Economy of Environmental Risk*, John Hopkins University Press.

Kimball, M., 1990, Precautionary savings in the small and in the large, *Econometrica* 58:53-73.

Lichtenstein, S., Slovic P., Fischhooff B., Layman M. and B. Combs, 1978, Judged Frequency of Lethal Events, *Journal of Experimental Psychology: Human Learning and Memory*, 4.

Margolis, H., 1996, *Dealing with Risks: Why the Public and the Experts Disagree on Environmental Issues*, Chicago Press.

Peltzman, S., 1975, The effects of automobile safety regulation, *Journal of Political Economy*, 83.

Pollack, R.A., 1995, *Regulating risks*, Journal of Economic Literature, 33.

Pollack, R.A., 1998, Imagined risks and cost-benefit analysis, *American Economic Review*, 88.

Portney, P.R., 1992, Trouble in Happyville, *Journal of Policy Analysis and Management*, 11.

Salanié, F. and N. Treich, 2002, Savings and the economics of self-control, mimeo.

Slovic, P., 1987, Perception of risk, *Science*, 236.

Thaler, R.H., 2000, From Homo Economicus to Homo Sapiens, *Journal of Economic Perspectives*, 14.

Tversky, A. and D. Kahneman, 1974, Judgment under uncertainty: Heuristics and biases, *Science*, 185.

Ulph, A. and D. Ulph, 1997, 'Global warming, irreversibility and learning', *E*conomic Journal 107:636-650.

Viscusi, K.P., 1998, *Rational Risk Policy*, Oxford University Press.

Viscusi, K.P, 2000, Risk Equity, *Journal of Legal Studies*, forthcoming.

Appendix

Proof of proposition 1: for any (r, s), one has from (7)

$$K(a, r, s) \leq K(a, r, r)$$

$$K(a, s, r) \leq K(a, s, s)$$

and by substracting we get

$$K(a, s, s) - K(a, r, s) \geq K(a, s, r) - K(a, r, r).$$

Denote $K_r(a, ., s)$ the slope of K in r (by linearity it is independent from r). We get

$$(s - r)(K_r(a, ., s) - K_r(a, ., r)) \geq 0$$

so that $K_r(a, ., s)$ is weakly increasing with s.

Then choose any $s_1 < s_2$. We have $K_r(a, ., s_2) \geq K_r(a, ., s_1)$, so that $K(a, r, s_2) - K(a, r, s_1)$ is weakly increasing with r. Apply at $r < s_1 < s_2$ to get

$$K(a, r, s_2) - K(a, r, s_1) \leq K(a, s_1, s_2) - K(a, s_1, s_1)$$

and we know that the right-hand side is non-positive. Therefore $K(a, r, s_2) \leq K(a, r, s_1)$, for $r < s_1 < s_2$. This shows the first part of the result. The case $r > s_2 > s_1$ is treated similarly.■

Proof of Proposition 2: the equivalence between i) and ii) was shown in the text. Now suppose ii). This system of inequalities is similar to (7). Because K_a is linear with respect to r, the same property as in Proposition 1 follows, so we obtain iii). Conversely iii) clearly implies ii) and iv). There only remains to show that iv) implies iii). Since K_s is linear in r, and is zero at $r = s$, we get

$$K_s(a, r, s) = K_s(a, 0, s) + rK_{rs}(a, r, s) = \frac{s - r}{s} K_s(a, 0, s).$$

This shows the desired result.■

Proof of Proposition 4: first recall that $b(a, (1-s)p + sq)$ is the unique maximizer of

$$\sum [(1 - s)p(x) + sq(x)]U(x, a, b)$$

so that it is characterized by

$$\sum[(1-s)p(x)+sq(x)]U_b(x,a,b(a,(1-s)p+sq))=0. \tag{15}$$

Differentiating with respect to s yields

$$\sum[q(x)-p(x)]U_b(x,a,b(a,(1-s)p+sq))+H(a,(1-s)p+sq))\frac{\partial}{\partial s}b(a,(1-s)p+sq)=0. \tag{16}$$

Using (15) once more we get

$$\sum[q(x)-p(x)]U_b(x,a,b(a,(1-s)p+sq))=-\frac{1}{s}\sum p(x)U_b(x,a,b(a,(1-s)p+sq)).$$

Therefore

$$\begin{aligned} K_s(a,0,s) &= [\sum p(x)U_b(x,a,b(a,(1-s)p+sq))].\frac{\partial}{\partial s}b(a,(1-s)p+sq) \\ &= \frac{1}{s}[\sum p(x)U_b(x,a,b(a,(1-s)p+sq))]'[H(a,(1-s)p+sq)]^{-1} \\ &\qquad [\sum p(x)U_b(x,a,b(a,(1-s)p+sq))]. \end{aligned}$$

Now saying that this quantity decreases with a, for any a, s, p, q, is equivalent to saying that

$$f(a,p,q)\equiv[\sum p(x)U_b(x,a,b(a,q))]'[H(a,q)]^{-1}[\sum p(x)U_b(x,a,b(a,q))]$$

is decreasing with a, for any a, p, q.

Let us first suppose that a is reduced by a difference in beliefs, so that iv) in Proposition 2 holds, and $f(a,p,q)$ is decreasing with a. Now suppose that $b(a,p)=b(a,q)$ at some (a, p, q). Then not only $f(a,p,q)=0$, but also $f_a(a,p,q)=0$ because all terms in the derivative vanish. Since anyway $f_a\leq 0$ by assumption, then it must be that f_a is at its maximum value, so that $f_{aa}=0$. Computing this second derivative, all terms vanish but

$$[\frac{\partial}{\partial a}\sum p(x)U_b(x,a,b(a,q))]'[H(a,q)]^{-1}[\frac{\partial}{\partial a}\sum p(x)U_b(x,a,b(a,q))]$$

so that this term must be zero. Since H^{-1} is negative definite, we get

$$\frac{\partial}{\partial a}\sum p(x)U_b(x,a,b(a,q))=0. \tag{17}$$

So we have proven that $b(a,p) = b(a,q)$ implies

$$\frac{\partial b}{\partial a}(a,q) = \frac{\partial b}{\partial a}(a,p).$$

Then there exists a vector $d(a,b)$ such that

$$b(a,p) = b_0 \Rightarrow \frac{\partial b}{\partial a}(a,p) = d(a,b).$$

Another manner to rephrase our result is the following. $b(a,p) = b(a,q)$ means that

$$\sum p(x) U_b(x, a, b(a,q)) = 0.$$

Define $G(a,q)$ as the $N \times X$ matrix with typical line $U_b(x, a, b(a,q))'$. Then we have

$$G(a,q)p = 0.$$

Now, from (17) we have shown that this implies

$$G_a(a,q)p = 0.$$

This implication is valid for any p. Therefore this means[16] that there exists a matrix $M(a,b)$ such that

$$G_a(a,q) = M(a,b)G(a,q).$$

This shows i).

There remains to show ii). From iv) in Proposition 2, we know that $K_a(a,0,s)$ must decrease with s. Notice that

$$K_a(a,0,s) = \sum p(x) U_a(x, a, b(a,(1-s)p + sq))$$

$$+ [\sum p(x) U_b(x, a, b(a,(1-s)p + sq))] . \frac{\partial b(a,(1-s)p + sq)}{\partial a}$$

$$= \left[\sum p(x)[U_a + U_b.d]\right](a, b(a,(1-s)p + sq)).$$

Differentiating with respect to s yields

$$\left[\sum p(x)[U_{ab} + U_{bb} d + d_b U_b]\right] . \frac{\partial}{\partial s} b(a,(1-s)p + sq) \leq 0.$$

[16] Lemma 1 below provides a formal proof of this point.

From i) the bracketted term is equal to $(M + d_b)U_b$; and the last term was computed in (16). Replacing we get ii).

Finally i) and ii) are clearly sufficient, as the last paragraph has shown.■

Lemma 1 *Suppose two matrices A and B are such that*

**) for all weight p, $Ap = 0 \Rightarrow Bp = 0$;*

***) there exists a weight q such that $Aq = 0$.*

Then there exists a matrix M such that $B = MA$.

Proof of Lemma 1: given the weight q, any vector v can be written $v = \alpha q + w$, with $\sum_x w(x) = 0$. Suppose that $Av = 0$. From **) we know that $Aq = 0$, so that we get $Aw = 0$. Now, for β small enough $p \equiv q + \beta w$ is a weight, and we have $Ap = 0$. From **), this implies that $Bp = 0 = Bq + \beta Bw$. From *) and **), we know that $Bq = 0$. Therefore we must have $Bw = 0$, and finally $Bv = \alpha Bq + Bw = 0$. Therefore we have shown that for any vector v, $Av = 0$ implies that $Bv = 0$. This shows the Lemma, from a standard property of matrixes.■

Proof of Proposition 5: the sufficiency part is proven in the text. Let us now show that ii) implies i). Consider the case when the decision-maker gets no information at all. His payoff when he chooses a is then $j(a, p)$. Now suppose that information is to arrive in the form of a binomial signal, yielding posterior beliefs q_1 with probability α and q_2 with probability $(1-\alpha)$. Bayesian revision of beliefs requires

$$p = \alpha q_1 + (1 - \alpha) q_2.$$

Then his payoff becomes

$$\alpha j(a, q_1) + (1 - \alpha) j(a, q_2).$$

Because any a can be a solution, ii) implies that the difference

$$\Delta(a, \alpha) \equiv \alpha j(a, q_1) + (1 - \alpha) j(a, q_2) - j(a, \alpha q_1 + (1 - \alpha) q_2)$$

is weakly increasing with a, for any α. Notice that $\Delta(a, 0) = 0$, for any a. Therefore, for Δ to be non-decreasing in a in the neighbourhood of $\alpha = 0$, it must be that its derivative at $\alpha = 0$ is non-decreasing with a. Use the enveloppe theorem to get

$$\Delta_\alpha(a,0) = j(a,q_1) - j(a,q_2) - \sum_x [q_1(x) - q_2(x)]U(x,a,b(a,q_2))$$

$$= j(a,q_1) - \sum_x q_1(x)U(x,a,b(a,q_2) = K(a,r,r) - K(a,r,s).$$

Because this difference must be increasing with a, we get i).■